HW #4 - Due in September 27. Do not copy from each other! I want to see how you are thinking!

1. Draw graphs of the relation between \( \sqrt{V_p/V_A} \) and \( \Theta \) for the fast mode, Alfvén, and slow mode for the two following cases:
   \[ \frac{\delta V_s^2}{V_A^2} = 2 \] and \[ \frac{\delta V_s^2}{V_A^2} = \frac{1}{2} \]. Assume \( \nu = \nu_a = 0 \).
   Identify the pure acoustic mode in \( \Theta = 0 \) and the magnetoacoustic in \( \Theta = \frac{\pi}{2} \). Calculate \( \sqrt{V_p/V_A} \) for \( \Theta = 0, \pi/4, \pi/2 \) and draw the rest.

2. Despite the fact that an interstellar cloud be predominantly made of neutral hydrogen and helium with \( n & 2 \times 10^{-20} \text{g} \), it posses sufficient number of ion pairs to support the three HHO modes. What are all the gas participating. A typical density of these clouds is \( n_0 = 6 \times 10^{-23} \text{cm}^{-3} \), temperature of 30 K and magnetic field of \( 5 \times 10^{-6} \text{G} \). Calculate \( V_p \) for the pure acoustic mode, the pure Alfvén mode, and the pure magnetoacoustic mode. Calculate the corresponding wave period (in years) for waves which wavelength is 0.5 pc.

3. Starting from the equation for \( V_A \) (see class notes), the phase velocity of the Alfvén wave \( \to \infty \) when \( n_0 \to 0 \). Calculate \( c^2 | \delta E/\delta t | / (1 \times 8) \) for a wave with \( V_A \ll c \), and show that \( \delta E/\delta t \) cannot be neglected in these circumstances. Come back to the basic equations and calculate \( V_p \) for an Alfvén wave (pure), including \( \delta E/\delta t \), and show that \( n_0 \) never exceeds \( c \).
   Formulate an inequality involving \( B_0, n_0, \) and constants, such that we can neglect \( \delta E/\delta t \). How big \( n_0 \) need to be in a dipole field of a neutron star (10^12 G in the surface) so we can neglect \( \delta E/\delta t \)?
4. The galactic, interstellar medium is supported against the gravity (\( g = 2 \times 10^{-9} \text{cm s}^{-2} \)) by a combination of turbulent stress, magnetic pressure due to a horizontal magnetic field and pressure of cosmic rays (the ones that couple to magnetic field and increase its effective pressure). The temperature of the gas (including turbulence) is equivalent \( T = 3600 \text{K} \), the average mass per particle (including helium) is \( 2 \times 10^{-4} \text{g} \), and a magnetic field \( B \), including the effect of cosmic rays, is equivalent to \( 5 \times 10^{-6} \text{G} \) in the \( z \) plane. \( z = 0 \) if \( z = B^2/8\pi \rho \) is independent of \( z \), calculate its value and the height \( H \) of the gas layer. Apply the von Neumann Taylor (describe in the class notes) to verify if that atmosphere is unstable in the \( z \) direction. If this is the case, calculate the minimum wavelength for which the instability occurs, and for a wavelength twice this value, calculate the growth time, expressed as \( \alpha^{-1} \) (in years).