\[
\dot{\Phi}(\phi) = \beta \cdot V_c \cdot \sqrt{x^2(1-x^2)} = 1 (5.5)
\]

Solving for \( x \):

\[
2(2-x^2) \dot{y}^2 + y[3(x^3-1)M^2 + 2(1+y^2)] - y(x^3 + 1)M^2 = 0 \quad (5.50)
\]

Given \( M \rightarrow \infty \), we find \( x \) and \( \dot{y} \).

Lecture 6 - Finite Slab and Wave (Shall Qu (via back + question))

- Project Paper (Until Friday) (WHERE ARE YOU IN THE APPLICATION PAPER?)
- HW #2 - due in 2 weeks, October 8

3.3 Shocks with \( \beta = 0 \)

Applied to weak or wave propagation parallel to \( \phi \)

So \( \beta = 0 \) in eq. (3.11) ->

\[
x = \frac{(1-x^2)M^2}{(x-1)}
\]

and \( x = 2 \alpha M^2(x-1) \) (see page 42)

- If \( M = 1 \)

\[
\dot{\Phi} = \frac{(8+1)}{(2+1)} \quad \text{or } \frac{(8+1)}{(2+1)} = 3 + 1
\]

\[
x = 2 - \frac{1}{x-1} = 1
\]

It's a very weak shock \( \phi \) with \( V_{es} = \frac{3}{5} V_c \)

- If \( M > 1 \)

\[
x = 8^{1/5} \quad \text{or } \frac{(8+5)}{(5+1)} = 4
\]

\[
\dot{\Phi} = \frac{3}{5} V_c
\]

Basically, any shocks move \( \phi \) to \( \Phi > \frac{3}{5} V_c \) (up to a certain speed).

Additionally, the shocks move \( \Phi \) to \( \frac{3}{5} V_c \) (up to a certain speed).
$x \rightarrow \frac{2\pi M^2}{3} \left( x \cdot \frac{\pi}{3} \right)$

\[ M = \frac{\pi}{2} \frac{M^2}{4} \]

$M^2 \rightarrow M^2 \frac{1}{16} \frac{\pi}{5} \frac{M^2}{4}$

$T_0 = (\pi/2)^2 \frac{M^2}{4} \rightarrow \frac{3}{2} \frac{M^2}{4} \frac{\pi}{5}$

$\frac{M^2}{4} \frac{\pi}{5}$

The thickness in the post-shock growth depends on $\nu_2$, $M^2 \frac{\pi}{5}$

As $\nu_2 = \nu_1 \rightarrow \frac{\pi}{5}$

$M^2 \frac{\pi}{5}$

The kinetic energy in the post-shock is $\frac{1}{16} \frac{\pi}{5} \frac{M^2}{4}$

of the pre-shock ($\nu_2$)

3.4 Shock wave: $\nu_2 \neq 0$

- Weak shock: $\gamma = 1$

$2\nu_2 (\nu_2 - \nu_1) \frac{M^2}{4} + \frac{2\pi}{16} \frac{\pi}{5} \frac{M^2}{4} = \nu_2 \frac{\pi}{5}$

$V^2 = \nu_2 \frac{\pi}{5} + \nu_1 \frac{\pi}{5}$

so $\nu_1 = 1$

The velocity found is the phase velocity of a magneto-sonic wave.

$M = \frac{\nu_2}{\nu_1} \frac{\nu_2}{\nu_1} = \left( 1 + \frac{\nu_2}{\nu_1} \right) \frac{M^2}{4}$

$M > 1 \rightarrow \text{short waves can open through B}$

For $M > 1$ as $\nu_2 \rightarrow 1 + 2\gamma_{\nu_2}$

$M^2 > 1 + \frac{2\gamma_{\nu_2}}{\nu_2}$

The growth of $B$ to reduce the pressure growth

(see page 23)
- Growth of temperature: growth of radiation
  - growth of $V$ 
  - loss of $V$
  - shocks through the magnetic field at least

If $M < 5$, then no shocks; if $B > 120 (B_1/\rho)^{1/2}$

$$p_1 + p_2 + p_3 = 0$$

- Dynamic pressure that produces the shock can become magnetic or thermal pressure
- Pressure of $B_1$ stress is absorbed by magnetic field without increase so much the temperature (as it will increase without $B$
- Magnetic clouds can take more "electricity"

35 Fermi-Abelaton

Cosmic rays 1914 - discovered with balloons
- particles with $\sim 10^8$ GeV
- $n(E) \propto E^{-p}$ (isotropy)
- protons, neutrons, or Fe, Fe, C, O, etc.

$$\log(n_0)$$

$$S = 2.6$$

Today we measure particles with $E \sim 10^{14-15}$ eV. We do not know
the origin. For particles with $E \sim 10^{15}$ eV its origin is unknown.
\[ \text{Average energy, Uranus, } E_{\text{rms}} = 10^{12} \text{erg/cm}^2 = 4.4 \text{eV/cm}^2 \]

\[ B_{\text{rms}} = 5 \times 10^{-6} \]

\[ \text{U} \text{b, } B_{\text{rms}} = 0.8 \times 10^{-6} \text{ erg/cm}^2 \]

The fact that Uranus \( = \text{U} \text{b, } B_{\text{rms}} \) (proportional) suggests that the cosmic rays and \( \text{U} \text{b, } B_{\text{rms}} \) magnetic field are interrelated.

Turbulence in \( E_{\text{rms}} \), magnetic field, cosmic rays, composition of energy, interaction.

We believe that cosmic rays are accelerated in shock waves originated in turbulence \( \leftrightarrow \text{SNRs} \) (shock waves), explosions of supernova.

Synchrotron radiation \( \gamma \text{ or } \gamma k^2 \) (relativistic electrons)

\( \gamma = \frac{E_{\text{rms}}}{E} \quad \text{if you want more detail, take} \]

\[ \gamma > 1 \quad \gamma > 2 - 3 \quad \text{@ Birkin \& Lightman 1988} \]

is produced by AGN, solar flares, and stars, relativistically.

We will study mechanisms of acceleration of particles with relativistic figures.

Vlasov Equation describes \( \mathbf{F} \) (direct analysis)

Phase space \( \mathbf{x}, p \)

Volume of a cell in space \( dV = d^3x \)

\[ \frac{df}{dt} + \nabla \cdot (f \mathbf{v}) + 2 \left( \nabla \cdot \mathbf{F} \right) = 0 \]

where

\[ \mathbf{F} = -\frac{df}{dp} \]
We exclude the contribution of close collisions between particles (binary collisions). The Lorentz force

\[ F = q(E + \frac{1}{c^2} E \times v) = \frac{d\vec{p}}{dt} \]

\[ p = m v \quad \therefore \quad \vec{v} = \frac{(1 - \frac{v^2}{c^2})^{-1/2}}{c^2} \]

\[ \frac{d\vec{F}}{dt} = \vec{F} = \frac{d}{dt}(q\vec{E}) + \frac{1}{c^2} \frac{d}{dt}(q(E \times \vec{v})) = \]

\[ = \frac{1}{c} \left[ \frac{\partial}{\partial t} (J \times \vec{B}) \right] - \frac{1}{c} J \cdot (\vec{B} \times \vec{E}) + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} \]

\[ + \vec{E} \cdot \frac{\partial}{\partial t} (J \times \vec{B}) = 0 \]

So

\[ \frac{d\vec{F}}{dt} + \frac{\partial}{\partial t} (J \times \vec{B}) = 0 \]

\[ \frac{\partial}{\partial t} \vec{v} = \vec{F} \cdot \vec{v} = 0 \]

If we apply these equations to "collisions" between particles and to magnetized clouds, it can be shown that the change in energy in a collision is small. Therefore, we can convert the Vlasov equation in an equation of Fokker–Planck. This is what we will do now.

- We assume that there is only one coordinate $x$, velocity $v$, and momentum $p$. Later, we will do a generalization to $3$ components.

If the probability that in a time $\Delta t$ the particles change their momentum by $\Delta p$ is given by $W(p, \Delta p)$, that depends only on their momentum in the present state and not at all their past history (Markovian process). The distribution of momenta in $t + \Delta t$ is

\[ \frac{dW}{d\vec{p}} \]
\[ f(p, x + v\Delta t, t + \Delta t) = f(p, x, t) + v\Delta t \frac{\partial f}{\partial x} (p, x, t) + \frac{\Delta t^2}{2} \frac{\partial^2 f}{\partial t^2} (p, x, t) \]

Expanding \( f \) and \( \psi \) in a Taylor expansion:

\[ f(p, x + v\Delta t, t + \Delta t) = f(p, x, t) + \Delta p \frac{\partial f}{\partial p} (p, x, t) + \frac{1}{2} \Delta p^2 \frac{\partial^2 f}{\partial p^2} (p, x, t) \]

Why \( \Delta x \) and \( \Delta t \) are first order and \( \Delta p \) is second order? Because only its increment, it's \( \Delta p \) itself.

\[ \psi(p - \Delta p, \Delta p) = \psi(p, \Delta p) - \Delta p \frac{\partial \psi}{\partial p} (p, \Delta p) + \frac{1}{2} \Delta p^2 \frac{\partial^2 \psi}{\partial p^2} (p, \Delta p) \]

[1] = \left[ \frac{\partial (A\psi)}{\partial p} \right] \times \left[ (3) \right] \]

\[ \frac{d}{dt} + v\Delta t \frac{\partial}{\partial x} + \frac{\Delta t^2}{2} \frac{\partial^2}{\partial t^2} = \left\{ \frac{d}{dt} (A\psi) \right\} + \mathcal{O}(\Delta p^2) \quad (3.14) \]

Once \[ d(A\psi) = 0 \]

\[ \int \frac{d}{dt} (A\psi) \ dx = \int \frac{d}{dt} (A\psi) \ dx = f(p, x, t) \]

\[ \frac{\langle \Delta p \rangle}{\Delta t} = \int \frac{d}{dt} (A\psi) \ dx \]

\[ \frac{\langle \Delta p \rangle^2}{\Delta t} = \int \frac{d}{dt} (A\psi) \ dx \]

(3.15) can be written as:

\[ e^{-v \Delta t} \delta x + v \Delta t \delta x = \int -f \Delta t \frac{\partial}{\partial t} \langle \Delta p \rangle + \frac{1}{2} f \Delta t^2 \frac{\partial^2}{\partial t^2} \langle \Delta p \rangle^2 \]

\[ + \cdots \]
\[ f_t + u \cdot \nabla f = \nabla \cdot (f \nabla p) + \frac{1}{2} \nabla \cdot \left( \nabla f \nabla \right) \]

The generalization in 3-D is

\[ f_t + v \cdot \nabla f = \nabla \cdot (f \nabla p) + \frac{1}{2} \nabla \cdot \left( f \nabla \frac{\partial f}{\partial p} \right) \]

Field: advection of \( f \) in the position space (with velocity \( v \))

Term (1): advection of \( f \) in the momentum space with a "velocity" \( \nabla f \) (\( v \) is replaced by \( \nabla f \) in the Navier-Stokes equation)

Term (2): diffusion of \( f \)
It can be demonstrated that the re-arranged motion can be neglected in each "collision" (what usually is justified because the ensemble of particles with which each particle is colliding has a much larger mass).

\[ \psi(p_i - \Delta p_i) = \psi(p_i - 4\Delta p_i) \]

Expanding in \( \Delta p \) (same as done before)

\[ \psi(p_i - \Delta p_i) = \psi(p_i + \Delta p_i) = \Delta p \frac{\Delta \psi}{\Delta p} + \frac{1}{2} (\Delta p)^2 \frac{\Delta^2 \psi}{\Delta p^2} \]

Integrating in \( \Delta p \) and multiplying by \( \Delta t \), we obtain:

\[ 1 = 1 - \Delta t \frac{2}{\Delta p} \left( \frac{\Delta \psi}{\Delta p} \right) + \Delta t \frac{1}{2} \Delta p^2 \frac{\Delta^2 \psi}{\Delta p^2} \]

So,

\[ \frac{2}{\Delta p} \left( \frac{\Delta \psi}{\Delta p} \right) - \frac{1}{2} \Delta p \frac{\Delta^2 \psi}{\Delta p^2} \bigg|_{\Delta t} = 0 \quad \text{in Zo} \]

In Zo:

\[ \frac{2}{\Delta p} \left( \frac{\Delta \psi}{\Delta p} \right) - \frac{1}{2} \Delta p \frac{\Delta^2 \psi}{\Delta p^2} \bigg|_{\Delta t} = 0 \quad \text{in Zo} \]

If we integrate this equation from \( \psi \) to \( \psi_i \) and we use the fact that the coefficient of \( \psi_i \) in Zo goes to zero when \( \Delta p \rightarrow 0 \)

\[ \frac{\Delta \psi}{\Delta p} = \frac{4}{2} \frac{\Delta p \Delta \psi}{\Delta t} \]
\[ \frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right) \]

\[ \frac{\partial}{\partial p} \left( \frac{\partial \phi}{\partial p} \right) \]

\[ \frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial x} \right) \]

\[ \frac{\partial}{\partial p} \left( \frac{\partial \phi}{\partial p} \right) \]

\[ \frac{\partial}{\partial t} \left( \frac{\partial \phi}{\partial x} \right) \]

Now the Fermi Theory (for the) He noticed that a magnetic field of \(10^{-6} - 10^{-8}\) G was necessary to confine the cosmic rays of the Galaxy. Carbon Rays: \( E \approx 10^8\) GeV \( \rightarrow 10^8\) eV = 10^8 \( \text{eV} \) 

\[ E = \gamma m_c \gamma = \gamma \frac{\gamma + \beta}{\gamma - \beta} = \gamma \frac{\gamma + \beta}{\gamma - \beta} \]

\[ \gamma = \sqrt{1 + \beta^2} \]

For \( E = 10^8\) GeV; \( \gamma \approx 10^8 \)

\[ R_{\text{eq}} = 10^8 \]

\[ r_{\text{eq}} = 3.10^8 \text{ cm} = 4.0 \] pc

He recognized that irregularities in the magnetic field associated with distribution in clouds in the interstellar medium. If \( B \) will be large inside other regions, and weaker in the external parts, a cosmic-ray moving along a line of force magnetic will tend to reflect when approaching the cloud. (Magnetic Venus - remember from past 2 of The Same?)

Reason:
For a given \( M \), we solve the equation

\[
2 \left( 2 - \frac{2}{\beta} \right) \frac{v}{v^*} + \left( \frac{2}{\beta} \right)^2 \left( \frac{M^2}{2} + 2 \left( 1 + \beta^2 \right) \right) y - \beta^2 (v^* + 1) = 0
\]

First, get \( y \) and then \( \beta \).

Remember, the above equation is good for supersonic shocks.

So for \( M = 5 \), \( \beta = 5/3 \)

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( x = \frac{P_a}{P_0} )</th>
<th>( \frac{y}{\beta^2} )</th>
<th>( \frac{x}{\beta^2} )</th>
</tr>
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<td>3.6</td>
<td>8.6</td>
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<tr>
<td>5</td>
<td>6.8</td>
<td>2.0</td>
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<td>2.5</td>
<td>1.5</td>
<td>1.3</td>
</tr>
<tr>
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<td>1.8</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>20</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

*Alert* - We will have shock if \( M^2 > 1 + 2/\beta \) or for this case \( 25 > 1 + 2/5 \)

\[
25 > 1 + 6/5
\]
The energy \( V_k = \text{cos}^2 \theta \) is conserved; where \( \theta \) is the angle between \( V \) and \( B \), "pitch angle".

\( V \) is constant so \( \text{cos} \theta = \text{const} \) and \( \theta = \pi \) (when \( \theta = 90^\circ \)). \( V_k = 0 \). \( V = V_k \) and the particle is reflected.

Applying the law of energy argues that when particles approach a cloud it will be reflected. The clouds are seen to move with \( \sim 10 \text{km/s} \).

Then, although the energy of the particle is conserved in the reference system of the cloud to the cloud itself (if the cloud is approaching) or decreases (if the cloud is distancing itself) in the reference system of the galaxy.

Ex. cloud with velocity \( V \).

Ref. system of the cloud:

\[ E_{\text{part}} = \text{ke} \]

Ref. of the galaxy:

\[ E_{\text{part}} \uparrow \text{if the cloud will approach} \]

\[ E_{\text{part}} \downarrow \text{if the cloud will distancing} \]

For the cloud

\[ V \rightarrow \quad \text{before} \quad \text{...} \]
So \( P_{\text{flow}} = \gamma m(v+v) \) \( \Delta p_{\text{flow}} = \gamma m(v+v) \)
\( P_{\text{coll}} = \delta m(v+v) \) \( \Delta p_{\text{coll}} = \delta m(v+v) \)

For the Galaxy:

\[ \vec{v} \quad \gamma \]
\[ \vec{v} \quad \gamma \]

\( \vec{v} \quad \gamma \)
\[ \vec{v} \quad \gamma \]

\[ M \quad \text{v} \]
\[ M \quad \text{v} \]

\[ \Delta p_{\text{coll}} = 2mv \]

\[ \text{If } \Delta p = 0 \quad \text{no } \Delta v \]

\[ \Delta p_{\text{coll}} = 2mv \]

\[ \text{If the cloud recedes} \]

\[ M \quad \rightarrow \quad v \]

\[ \Delta p_{\text{coll}} = m(v+v) = 0 \]

Consider several clouds:

\[ \text{risk of gravitational collisions} \]

\[ \text{approaching} \]

\[ v_{\text{c}} = v+v \]

\[ \text{receding} \]

\[ v_{\text{c}} = v-v \]

\[ \text{If } v_{\text{c}} > v \text{, there will be a} \]
And in 3.5:
\[ D_{ij} = \frac{1}{2} \left< \frac{\partial \mathbf{p}_i \cdot \mathbf{p}_j}{\partial \mathbf{p}_i} \right> - \frac{4}{3} \left< \mathbf{v} \right> \mathbf{v}_i \delta_{ij} \]

for relativistic particles. The Söderbergh-Planck equation then has to be
\[ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{f} = 2 \left( \left< \mathbf{v}^2 \right> \frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \frac{\partial}{\partial \mathbf{v}} \left< \mathbf{v} \right> \right) + \left< \mathbf{v} \right> \frac{2}{3} \left( \frac{\partial \mathbf{f}}{\partial \mathbf{v}} \right) \]  
(3.83)

Blandford and Gersh (1982) show that this equation is also true for \( \mathbf{v} \mathbf{c} \) \( \rightarrow \) you just need to substitute \( c \) by \( \mathbf{v} \)