2D MHD simulation of the magnetic dipole tilt and IMF influence on the magnetosphere

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Abstract

Global magnetohydrodynamic (MHD) models, developed to solve the magnetospheric configuration and provide a self-consistent picture of the solar wind-magnetosphere interaction, has been successfully used for the past decades. Here, we modified a 2D MHD code to include a magnetic dipole tilt angle different from zero. Our goal is to investigate the formation of the earth’s magnetosphere for different magnetic dipole tilt and interplanetary magnetic field (IMF) configurations. Our results for the effects of the magnetic dipole inclination on the magnetosphere structure, considering null IMF and three different tilt angles, are consistent with previous works. The position of the magnetospheric current system is dependent on the magnetic dipole tilt. Introducing IMF into the system intensifies or decreases those current systems. The variation of the density current intensity occurs for opposite directions for northward and southward IMF condition. Those intensifications are coincident with regions predicted by the reconnection theory.

Keywords: MHD simulation, magnetosphere formation, dipole tilt
1. Introduction

Space weather refers to conditions on the Sun and solar wind, magnetosphere and ionosphere that can influence the performance and reliability of ground-based technological systems and can affect the human life and health (Baker 1998, Siscoe 2000, Gombosi et al. 2004, Echer et al. 2005, Schwenn 2006). The size of the magnetosphere is determined by the balance between the solar wind dynamic pressure and the pressure exerted by the magnetosphere, mainly due to its magnetic field (Kivelson and Russell, 1995). The shape of the magnetosphere is additionally influenced by the drag of the solar wind, or tangential stress, on it. This drag is predominantly caused by the mechanism known as magnetic reconnection, in which the magnetic field of the solar wind links to the magnetic field of the magnetosphere (Russell, 1991). This interaction between the solar wind and the planetary magnetospheres is not easily understood. The heliosphere and the magnetosphere are vast regions of space with relatively few measurements due to the spacecraft trajectories limitation (Gombosi et al., 2004).

Global magnetohydrodynamic (MHD) model have been developed to provide a self-consistent picture of the solar wind-magnetosphere interaction process (Walker and Ogino, 1989). The first global MHD simulation of the Earth’s magnetosphere was developed by Lebouef et al. (1978), in two dimensions. They observed magnetic field reconnection in southward e northward interplanetary magnetic field (IMF) conditions and turbulent magnetosheath and magnetopause. The same group developed the first three-dimensional
(3D) MHD code time dependent. In 3D, the solar wind magnetic field lines can avoid the nose and tail reconnection altogether by flowing around the magnetosphere in the magnetosheath. However, that situation is not possible in their model due to the high resistivity of the flow which is opposite to real magnetosphere, an almost null resistivity condition. The 3D code showed the same topology of the 2D simulation.

Lyon et al. (1980) developed a minimally diffusive 2D MHD code of the Earth's magnetosphere and the solar wind interaction, which solves the MHD equations of a single flow. Their results showed structures as the bow shock, the magnetopause and a long magnetotail. Their code, using Leapfrog time-integration method, was used to simulate events, as for example, substorms in the Earth's magnetosphere and the results were coherent with an empiric model of substorms.

Two-dimensional models of the Earth's magnetosphere are not as complete as the three-dimensional ones due to 3D nature of the solar wind and the all system. For this reason, Brecht et al. (1981) developed a three-dimensional code including the IMF $y$ component (east-west) and $z$ component (north-south) to study the effects of this magnetic field on the earth magnetotail configuration. They observed a great correspondence between the existence of the $y$ component of the IMF and the rising of the $y$ component of the magnetotail magnetic field.

Wu (1984) worked on a MHD simulation of the magnetic dipole inclination influence on the magnetospheric structure. Besides the surface currents observed in Chapman-Ferraro model (a vacuum model), currents were also observed inside the magnetosphere (Wu, 1983). In addition to the tail cur-
rent sheet, the MHD model created by Wu (1984) predicts the existence of a
current sheet above the cusp region, which was called the cusp current sheet.
The tail and the cusp current sheet showed to be a function of the dipole tilt.
Zhou et al. (1999) also worked on dipole tilt influence but in this case related
to the polar cusp location. They observed when dipole tilts more toward the
Sun, the cusp moves more poleward to higher latitude positions.

Ogino et al. (1985) used a three-dimensional code to solve the magneto-
spheric configuration when the IMF have north-south and east-west compo-
nents. The generation of field-aligned currents, which are important in the
energy transfer between the solar wind and the magnetosphere-ionosphere,
also became a matter of study (Ogino et al., 1986b). Walker and Ogino
(1989) showed the dependence of the magnetospheric shape when northward
or southward IMF is introduced into the system in 3D simulation, consider-
ning the steady-state configuration of the magnetosphere in the absence of the
IMF. They first modeled the magnetosphere in the absence of the IMF and
found a slow reconnection in the plasma sheet. Their results indicated that
the presence of the northward magnetic field in the plasma sheet stabilizes
the tail.

The solar wind dynamic pressure and the IMF control the Earth’s mag-
netosphere and also other planetary magnetospheres. MHD simulations have
been performed for magnetospheres besides the Earth as well. Walker et al.
(2001) studied the effects of the solar wind dynamic pressure and the IMF on
the configuration of the Jovian magnetosphere. When the dynamic pressure
increases, the bow shock and the magnetopause move towards Jupiter and
when it decreases, the boundaries move farther from Jupiter. For northward

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IMF the boundaries move toward Jupiter and for southward IMF it moves away from it.

In this work, we have modified the two-dimensional MHD code developed by T. Ogino, 1986 (Matsumoto and Omura, 1993, Ogino et al., 1985, 1986b,a, Walker and Ogino, 1989, Ogino et al., 1992, Walker et al., 2001). The main modification is the magnetic dipole tilt angle inclusion for the Earth’s magnetic field. The magnetic dipole tilt angle is defined by the angle between the Earth’s north dipole axis and the GSM z-axis (Zhou et al., 1999). Our goal is to investigate the magnetosphere structure for different magnetic dipole tilt angle values. This allow us to understand different planetary magnetospheres besides the Earth. Such magnetospheric configurations are also submitted to different IMF conditions. Our results for the effects of the magnetic dipole inclination on the magnetosphere under conditions of null IMF are coherent to those presented by Wu (1984). We have also observed the effects of northward and southward IMF for different magnetic dipole tilt conditions.

In fact, dipole tilt influence on the magnetospheric dynamics study may be applied to other planets. Neptune presents a magnetosphere similar to the Earth, exhibiting regions like bow shock, magnetopause and magnetotail. Despite of that, Neptune magnetic dipole axis is tilted 47° in relation to the planet spin axis, which, in turn, is also inclined by 29° from the orbital plane (Russell 1991, Kivelson and Russel 1995). Uranus has also a comparable magnetosphere though its magnetic dipole is tilted 59° from the rotation axis (Russell 2001, Shirley and Fainbridge 1997). In this case, the spin axis is found to be nearly the Sun-Uranus orbit
plane what makes solar wind direction to be roughly perpendicular to the magnetic axis, as it is observed in Mercury, Earth, Jupiter and Saturn.

The magnetic dipole tilt variation analysis may also be even applied to the Earth. Studies of remanent magnetization in fossil rocks, known as paleomagnetism, give evidences that the inner geomagnetic field reverts its poles over long timescales (Campbell 1997, Parkinson 1983).

This paper is organized in four sections. Section 2 is about the 2D MHD model used. Section 3 describes a discussion of the results which is divided in two subsections; one (3.1) is about the validation of the modified code and the other one (3.2) is actually a discussion about the results. Section 4 brings conclusions of this work.

2. The two-dimensional MHD model

The magnetohydrodynamic equations combine Maxwell’s equations, used to describe electromagnetic fields, and the conservation of hydrodynamic laws.

The MHD equations in this model are normalized according to some typical parameters such as radius of the Earth, \( x_s = R_E = 6.37 \times 10^6 \) m, the magnetic field of the Earth at one earth radius on the equator \( B_s = 3.12 \times 10^{-5} \) T, the density of the ionosphere, \( \rho_s = mn_s(n_s = 10^{10} \text{m}^{-3}) \), the Alfvén velocity at one earth radius, \( v_s = B_s/(\mu_0 \rho_s)^{\frac{1}{2}} = 6.80 \times 10^6 \) m/s, the Alfvén transit time, \( t_A = R_s/v_s = 0.937 \) s, pressure, \( p_s = \rho_s v_A^2 = B_s^2/\mu_0 = 7.75 \times 10^{-4} \) N/m², current density, \( J_s = B_s/\mu_0 x_s = 3.90 \times 10^{-6} \) A/m², and the Alfvén acceleration, \( a_A = v_s/t_s = 7.26 \times 10^6 \text{m/s}^2 \) (Matsumoto and Omura,
Such equations, already normalized, are displayed below (Matsumoto and Omura, 1993):

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) + D \nabla^2 \rho, \quad (1)
\]

\[
\frac{\partial \mathbf{v}}{\partial t} = - (\mathbf{v} \cdot \nabla) \mathbf{v} - \frac{1}{\rho} \nabla p + \frac{1}{\rho} \mathbf{J} \times \mathbf{B} + \mathbf{g} + \frac{1}{\rho} \Phi, \quad (2)
\]

\[
\frac{\partial p}{\partial t} = - (\mathbf{v} \cdot \nabla) p - \gamma p \nabla \cdot \mathbf{v} + D_p \nabla^2 p, \quad (3)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad \text{and} \quad (4)
\]

\[
\mathbf{J} = \nabla \times (\mathbf{B} - \mathbf{B}_d), \quad (5)
\]

where \( \rho \) is the plasma density, \( \mathbf{v} \) is the flow velocity, \( D \) is the density diffusion coefficient, \( p \) is the plasma pressure, \( \mathbf{J} \) is the current density, \( \mathbf{B} \) is the magnetic field, \( \mathbf{g} \) is the gravity acceleration, \( \Phi \equiv \mu \nabla^2 \mathbf{v} \) is the viscosity, \( \gamma = 2 \) is the ratio of specific heats corresponding to degree of freedom of the system, \( D_p \) is the pressure diffusion coefficient, \( \eta = \eta_0 (T/T_0)^{-3/2} \) is the resistivity, where \( T = p/\rho \) (Matsumoto and Omura, 1993), and \( \mathbf{B}_d \) is the dipole magnetic field.

The viscosity and the diffusion terms are used to reduce the MHD fluctuations generated by unbalanced forces at the calculation beginning. Such oscillations tend to occur at the bow shock and at the magnetopause with the same scale as the grid spacing (Matsumoto and Omura, 1993).
The simulation model equations are solved as an initial value problem. A steady state ionosphere is used in the neighborhood of the Earth. Such ionospheric conditions are given by (Matsumoto and Omura, 1993): density,

\[
\rho_0 = \xi^{-2}, \quad \text{if} \quad \rho_0 \geq 0.2\rho_{sw} \quad \text{and} \\
\rho_0 = 0.2\rho_{sw}, \quad \text{if} \quad \rho_0 < 0.2\rho_{sw} \quad ; \quad (6)
\]

pressure,

\[
p_0 = p_{00}\xi^{-1}, \quad \text{if} \quad p_0 \geq p_{sw} \quad \text{and} \\
p_0 = p_{sw}, \quad \text{if} \quad p_0 < p_{sw} \quad ; \quad (7)
\]

gravity acceleration,

\[
g = \frac{g_0}{\xi^2}(x, z). \quad (8)
\]

and the magnetic field with dipole tilt angle inclusion,

\[
B_d = \frac{1}{\xi^4}[-2xz\cos\theta + \sin\theta(x^2 - z^2), 2xz\sin\theta + \cos\theta(x^2 - z^2)]. \quad (9)
\]

The normalized equations are solved using the two-step Lax-Wendroff method (Potter, 1973). The numerical stability condition of the difference scheme is the Courant-Friedrichs-Lewy law. Such condition is given by

\[
v_g^{\text{max}} \frac{\Delta t}{\Delta x} < 1, \quad \text{where} \quad v_g^{\text{max}} \quad \text{is the maximum Alfvén velocity in the calculation domain (Walker and Ogino, 1989).}
\]

The 2D simulation plane of the solar wind-magnetosphere interaction is shown in Figure 1, with \(x_0 \leq x \leq x_1\) and \(-z_0 \leq z \leq z_0\), in the cartesian
coordinate system. The center of the Earth is situated at \((x, z) = (0, 0)\).

The solar wind flows into the simulation plane along the \(x\) direction, through the boundary at \(x = x_0\).

The original code (Walker and Ogino, 1989) computes all the quantities for only half of the simulation box. As the magnetic dipole inclination brings asymmetry into the system, the calculation had to be done over all the box simulation, that is, for both hemispheres. Thus, the modifications implemented in the original code are: (1) enlargement of the box simulation for computing the quantities in both hemispheres; (2) changing the boundary conditions; (3) magnetic dipole tilt inclusion.

The modified boundary conditions are given by:

1. fixed boundary: \(\phi\) is constant in \(x = x_0\);
2. free boundary: \(\partial \phi / \partial x = 0\) in \(x = x_1, z = -z_0\) and \(z = +z_0\);
3. all physical quantities are fixed for \(\xi = (x^2 + z^2)^{1/2} \leq \xi_a (= 16, 0)\), where \(\xi_a\) is the ionospheric boundary.

The internal quantity \(\phi_{in}\), which are ionospheric values in the initial state, and the external quantities \(\phi_{ex}\) are connected at each time step by the introduction of a smooth function (Matsumoto and Omura, 1993).

### 3. Results and discussion

In this work, we investigate the formation of the magnetosphere under three different magnetic dipole inclinations \((0^\circ, 45^\circ, 90^\circ)\). We also consider three different conditions for the \(z\) component of the IMF, null, positive with northward direction \((+5 \text{ nT})\) and negative with southward direction \((-5 \text{ nT})\).
The IMF and the dipole tilt quantities influence the magnetosphere structure, more specifically its current systems.

The solar wind velocity parameter used here is referent to a quiet day condition with the value of 300 km/s. The spatial grid is defined by $N_x = N_z = 102$ and the mesh size is $\Delta x = \Delta z = 2, 0R_E$ and $\Delta t = 15s$.

At the initial state of the simulation, there is no solar wind in the box and the earth dipolar magnetic field is symmetric in relation to the dipole axis. Next, the solar wind under the presence or absence of the interplanetary magnetic field flows into the simulation box interacting with the earth dipole magnetic field. In turn, the earth magnetic field becomes asymmetric. The dayside is compressed and the nightside is elongated. The long region on the nightside is called magnetic tail. Current systems are observed inside the magnetosphere and their relation with the magnetic dipole tilt angle and the IMF has been investigated.

3.1. Adapted Code validation

First of all, it is important to verify if the modified code keeps yielding coherent outcome. Wu (1984) worked on magnetic dipole tilt effects on the magnetosphere current structure under null IMF condition. The magnetic dipole tilt values he used are 0° (dipole axis perpendicular to the solar wind flow), 45° and 90° (dipole axis parallell to the solar wind flow).

Figure 2 shows the current density intensity obtained by the adapted Ogino’s code. The panels refer to magnetic dipole angle values of 0° (top panel), 45° and 90° (bottom panel) under null IMF condition. In Figure 2, blue regions represent negative values indicating the current is directed out of the plane. Yellow and red colors represent positive values with the direction
coming into the plane. The black lines correspond to the magnetic field.

The magnetopause current sheet, the cusp current sheet and the beginning of the tail current sheet have been observed under conditions of null IMF and null magnetic dipole tilt. In Figure 2, we also observe that as the dipole tilt angle increases the current systems change their position. At the bottom panel, for the extreme case of 90° magnetic dipole tilt, it is noticed two current sheets at the magnetopause and two located in the magnetotail with opposite directions. The results presented in Figure 2 agree with those presented by Wu (1984). Such results are a strong evidence that the modifications have been implemented correctly, therefore validating our model.

3.2. Magnetic dipole tilt angle and IMF effect on the magnetospheric current configuration

In addition to the magnetospheric current configurations presented previously for null IMF and the three values for the magnetic dipole tilt angle, the same study has been done for northward and southward IMF condition.

Figure 3 shows the magnetic field lines for northward and southward IMF condition when the solar wind flow is perpendicular to the magnetic dipole axis. In terms of magnetic field line topology, it is possible to observe the difference in the magnetosphere structure for different IMF conditions. For northward IMF (top panel), the reconnection of the magnetic field lines is related to a current system coming into the plane close to both cusps. For southward IMF (bottom panel), the magnetic reconnection is expected to occur at the dayside and in the tail. Figure 3 presents only the dayside phenomenon. A current system appears coming out of the page during the magnetic reconnection.
Even though, the magnetic field line topology in Figure 3 makes easier our understanding, we think it is more appropriate to study the current systems instead. For this reason, we have investigated the current density varying with the magnetic dipole tilt angle and the IMF direction.

Figure 4 displays the current density intensity for northward and southward IMF conditions and for null magnetic dipole tilt which means that the magnetic dipole axis is perpendicular to the solar wind flow direction. Comparing the northward magnetic field with the null IMF condition in Figure 2, it is seen an intensification in terms of current density coming into the plane and a decrease of the current density in the opposite direction. On the other hand, for the southward IMF, there is an intensification of the current density coming out of the plane and a decrease in the opposite direction. The intensificated current areas are coincident with reconnection regions according to the magnetic reconnection theory. For the northward IMF, the reconnection regions are located at the cusps. Magnetic islands are formed and dragged with the solar wind in the tail direction. For the southward IMF, such areas are situated at the front of the magnetosphere and in the magnetotail.

Figure 5 shows the magnetic dipole tilted in an angle of 45°. For the northward IMF, the current density directed into the plane is intensificated close to the tilted cusps. Under southward IMF condition, it occurs for the current density directed out of the plane. Comparing to the null magnetic dipole tilt condition, the intensified regions in this case are also tilted.

The last condition is the magnetic dipole tilted in an angle of 90°, where the magnetic dipole axis is parallel to the solar wind flow (see Figure 6). The
same characteristics mentioned previously are observed in this case. For the northward IMF, the current density directed into the plane is more intense for regions located in front of the magnetosphere in the northern hemisphere and in the tail of the southern hemisphere, still close to the cusp which is tilted now. For the southward IMF, the opposite situation occurs, the current density directed out of the plane is intensified in the north tail and in the south front of the magnetosphere. What comes out in this magnetic dipole tilt value is the symmetry verified for the current configuration when both IMF conditions are compared.

Although 2D MHD code brings a reasonable description of the magnetospheric dynamics, it has not been considered complete (Brecht et al., 1981). Solar wind exhibits three-dimensional nature, particularly the IMF (Akasofu et al. 1978, Ogino et al. 1986b, Ogino et al. 1992). It is known that IMF x and y-component influence strongly the magnetotail structure and dynamics (Ogino et al. 1985, Ogino et al. 1986a). Hence, the interaction between solar wind and earth’s magnetic field may be pictured more accurately in the 3D system.

4. Conclusions

The main purpose of this work is to investigate the magnetospheric structure due to different magnetic dipole inclinations and different IMF configurations, using a modified version of the code created by T. Ogino (Matsumoto and Omura, 1993). Our modified version of the code includes magnetic dipole tilt angle and computes the physical quantities for both hemispheres due to
the asymmetry introduced into the system. The boundary conditions were also changed. Our results for the magnetospheric current configurations are in agreement with the results presented by Wu (1984).

Our results show that the cusp currents become a magnetopause and a tail current at the extrem case of 90° magnetic dipole tilt under null IMF. The current systems position is in fact dependent on the magnetic dipole tilt angle, confirming Wu (1984) results. All of this is possible in the MHD model because it allows currents inside the magnetosphere what does not happen in the Chapman Ferraro model, which considers a vacuum magnetosphere (Wu, 1984).

The magnetospheric current density directed into the plane is intensified when the IMF is northward and the current density with the out-of-the-plane direction when the IMF is southward, for all dipole tilt values. These results are coherent with the magnetic field topology scheme presented in Figure 3 for null magnetic dipole tilt. The earth magnetic field lines are connected to the IMF lines at the dayside and in the tail for southward IMF condition. For the northward IMF, the magnetic field lines connection is seen at the cusps. The current density intensification regions are coincident with those reconnection areas predicted by the magnetic lines scheme in Figure 3.

Besides the ionosphere and the magnetosphere conditions are settled for the Earth in this work, a similar study could be extended to other Earth-like planets, since the magnetic dipole tilt angle is included. Even though IMF exhibits three-dimensional nature affecting, for instance, the magnetotail structure and dynamics, a 2D description is still acceptable (Wu, 1984). It is believed that magnetosphere is mostly dependent
on southward and northward IMF, and hence, a 2D approximation brings a global picture of the magnetosphere (Kivelson and Russel, 1995). But solar wind can not flow completely around the Earth in the 2D system. Therefore, specific and accurate results may be obtained from a 3D code.

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Figure 1: 2D cartesian simulation box. The solar wind, represented by solid arrows, flows into the plane in the $x$ direction. The Earth is centered at $(x, z) = (0, 0)$. The dashed lines represent magnetic field lines of the Earth’s magnetosphere.
Figure 2: **Normalized current density** for magnetic dipole tilts of 0° (top panel), 45° (center panel) and 90° (bottom panel) under null IMF condition. These results are coherent with those presented by Wu (1984).
Figure 3: Magnetic field line topology for northward (top panel) and southward (bottom panel) IMF condition and solar wind flow perpendicular to the magnetic dipole axis configuration.
Figure 4: Normalized current density for null magnetic dipole tilt (magnetic dipole axis perpendicular to the solar wind flow) under northward IMF (top panel) and southward IMF (bottom panel) condition.

Figure 5: Normalized current density for 45° magnetic dipole tilt under northward IMF (top panel) and southward IMF (bottom panel) condition.
Figure 6: **Normalized current density** for 90° magnetic dipole tilt (magnetic dipole axis parallel to the solar wind flow) under northward IMF (top panel) and southward IMF (bottom panel) condition.