(a) We denote the state by $Y_m$

$$ L_m = \frac{1}{2} (l+L-) $$

therefore,

$$ \langle L_m \rangle = \langle Y_m^m | L_m | Y_m^m \rangle $$

$$ = \langle Y_m^m | \frac{1}{2} (l+L-) | Y_m^m \rangle $$

$$ = \frac{1}{2} \langle Y_m^m | \text{constant} | Y_m^m \rangle + \text{constant} \langle Y_m^m | Y_m^m \rangle $$

$$ = \frac{1}{2} \langle Y_m^m | \text{constant} | Y_m^m \rangle + \frac{l}{2} \langle Y_m^m | \text{constant} | Y_m^m \rangle $$

$$ = \frac{1}{2} (\text{constant}) \langle Y_m^m | Y_m^m \rangle + \frac{l}{2} (\text{constant}) \langle Y_m^m | Y_m^m \rangle $$

$$ = 0 + 0 $$

(From $Y_m^m$ is orthogonal to $Y_m^m$)

Hence, it is proved that $\langle L_m \rangle = 0$

In a similar way using $L_y = \frac{l+L-}{m}$, we can show $\langle L_y \rangle = 0$

(b) We will prove it in 2 ways: First using Symmetry, and then using a straightforward computational method.

**Using Symmetry:**

From Symmetry $\langle L_x^2 \rangle = \langle L_y^2 \rangle$.

Then $\langle L_z^2 \rangle = \langle L_x^2 \rangle + \langle L_y^2 \rangle + \langle L_z^2 \rangle = \frac{1}{2} \langle L_x^2 \rangle + \langle L_y^2 \rangle$.

Using $\langle L_z^2 \rangle = 2L_z^2$.
Given \( (\alpha) \) & \( (\beta) \), we can write

\[
\begin{align*}
2 \langle \lambda \mu \rangle + \langle \lambda \nu \rangle &= \frac{k^2}{2} (\lambda + 1) \\
2 \langle \lambda \nu \rangle &= \frac{k^2}{2} (\lambda + 1) - \langle \lambda \nu \rangle \\
\text{hence, it is found that} &\quad \langle \lambda \nu \rangle = \frac{\frac{\lambda^2}{2} (\lambda + 1) - \lambda k^2}{2} \\
\end{align*}
\]

**Direct Computational Method**

First we will compute:

\[
\begin{align*}
\langle \lambda \nu \rangle &= \langle \lambda \nu | \lambda \nu \rangle \\
&= \langle \lambda \nu | \lambda \nu \rangle [\text{using the hermiticity property of } \lambda \nu] \\
\end{align*}
\]

Now,

\[
\begin{align*}
\langle \lambda \nu \rangle &= \frac{1}{2} \left[ \lambda (\lambda + 1) \right] \mu_{\lambda \nu} \\
&= \frac{1}{2} \left[ \lambda (\lambda + 1) + \mu_{\lambda \nu} \right] \\
&= \frac{1}{2} \left[ \lambda + \mu_{\lambda \nu} \right] \\
\end{align*}
\]

where

\[
\begin{align*}
\mu_{\lambda \nu} &= \lambda (\lambda + 1) + \frac{1}{2} \mu_{\lambda \nu} \\
\end{align*}
\]
\[
\langle w^2 \rangle = \langle \ln y_n^m | \ln y_n^m \rangle \\
= \frac{1}{y^2} \left( \langle A_1 y_n^{m+1} + A_2 y_n^{m-1} | A_1 y_n^{m+1} + A_2 y_n^{m-1} \rangle \right) \\
= \left( \frac{\gamma}{\cosh^2 \phi} \right) \left[ \langle \ln y_n^{m+1} | \ln y_n^{m+1} \rangle + \langle \ln y_n^{m-1} | \ln y_n^{m-1} \rangle \right] + \langle \ln y_n^{m+1} | \ln y_n^{m+1} \rangle + \langle \ln y_n^{m-1} | \ln y_n^{m-1} \rangle \\
= \left( \frac{\gamma}{\cosh^2 \phi} \right) \left[ \left| A_1 \right|^2 + \left| A_2 \right|^2 \right] \quad \text{[We have used the orthonormality conditions of } y_n^m \text{]} \\
= \left( \frac{\gamma}{\cosh^2 \phi} \right) \left[ \zeta^2 (l-m)(l+m+2) + \zeta^2 (l+m)(l-m+2) \right] \\
= \left( \frac{\zeta^2}{\cosh^2 \phi} \right) \left( l^2 - m^2 + l \cdot m + \frac{l}{2} \cdot m (l-m+1) \right) \\
= \left( \frac{\zeta^2}{\cosh^2 \phi} \right) \left( 2l^2 - m^2 + 2l \right) \\
= \left( \frac{\zeta^2}{\cosh^2 \phi} \right) \left( l(l+1) - m^2 \right) \quad \text{(11)}
\]

Next, we write
\[
\langle \ln \gamma \rangle = \langle \ln y_n^{m+1} \rangle = \langle \ln \gamma \rangle + \langle \ln y_n^m \rangle + \langle \ln y_n^{m-1} \rangle \\
\Rightarrow \langle \ln \gamma \rangle = \frac{1}{2} \zeta^2 (l+1) = \langle \ln \gamma \rangle - \frac{1}{2} \zeta^2 (l+1) \\
= \frac{1}{2} \zeta^2 (l+1) - \frac{\gamma}{\cosh^2 \phi} \left( l(l+1) - m \right) - \mu \zeta^2 \\
= \frac{1}{2} \zeta^2 \frac{(l-1)^2 - m^2}{(l+1)^2 - m^2} \\
= \zeta^2 \frac{l^2 - m^2}{2} = \frac{\zeta^2}{2} \left( l(l+1) - m \right) \quad \text{(12)}
\]
hence, we have \[ \langle \text{lin}^2 \rangle = \langle y^2 \rangle = \frac{\int x^2 L(x^2) - x^2 y^2}{\int} \]

Prob. 2.35

(a) No, it is not a valid Hamiltonian form. Because \( L_x, L_y, L_z \) are not canonical momenta. Canonical variables satisfy the relation \( \{ P_i, \Pi_j \} = 0 \)

(b) The appropriate Hamiltonian is given as

\[ H = \frac{x^2}{2a^2} + \frac{y^2}{2b^2} \]

eigenvalues

\[ \frac{L^2}{2} \left( 1 + \frac{x^2}{a^2} \right) + \frac{m^2}{2b^2} \]

eigenfunctions corresponding to \( (l|m) \) is

\[ \psi_{l m} \psi_{-m} \]

(c) For a given eigenenergy there are two possible eigenfunctions as we saw above. For the molecule \( \text{H}_2 \). The degeneracy of the eigenenergy is also twofold.

(d) The ground state occurs for \( l=0, m=0 \)

The energy eigenvalue is 0.

For \( l=0 \) we use the relationship

\[ \frac{\omega^2}{2} = -\frac{1}{2} + \frac{1}{2} \frac{\omega^2}{2} \]

In the book it is written: \( \frac{1}{2} - \frac{1}{2} = -\frac{1}{2} \), which is wrong.
We have,
\[ e = \frac{h^2}{2} \left( \frac{e}{m} - \frac{1}{2} \right) + \frac{m^2 v^2}{2m} \]

\[ E_g = 0, \quad l = 0, \quad m = 0 \]

Next, we compute \( E \) for \( l = 2, \quad m = 0 \)

\[ E_2 = \frac{2h^2}{2m} \quad \text{(a)} \]

We also compute \( E \) for \( l = 2, \quad m = \pm 1 \)

\[ E_2 = \frac{2h^2}{2m} + \frac{h^2}{2m} \cdot \frac{E_g}{2} \]

\[ = \frac{2h^2}{2m} + \frac{h^2}{2m} \cdot \left( \frac{1}{2} - \frac{1}{m} \right) \]

\[ = \frac{2h^2}{2m} + \frac{h^2}{2m} \cdot \frac{1}{2m} \quad \text{(using)} \quad \frac{1}{2m} = \frac{1}{2} + \frac{1}{m^2} \]

\[ = \frac{2h^2}{2m} + \frac{h^2}{2m} \cdot \frac{1}{2m} \quad \text{(b)} \]

Hence, from (a) and (b), we see,

\[ E_2 > E_1 \]

Hence, the 2nd excited state occurs for

\[ l = 2, \quad m = \pm 1 \]

\( \epsilon \), the frequency of the emitted photon

\[ \frac{\Delta E}{\hbar} = \frac{E_2 - E_g}{\hbar} = \frac{1}{h} \left( \frac{2h^2}{m} + \frac{h^2}{2m} \cdot \frac{E_g}{2m} \right) \quad \text{[Eq. (a)]} \]

The energy of the incident photon should be just \( (E_2 - E_g) \).
\begin{equation}
\psi(\tau, 0) = \frac{e^{-\frac{i\tau}{2}}}{\sqrt{2}} \left( e^{i\frac{\pi}{4}} m^1 \right) \exp \left[ i (J_1 + \bar{J}_1) \right]
= \frac{e^{-\frac{i\tau}{2}}}{\sqrt{2}} \left( e^{i\frac{\pi}{4}} m^1 \right) \exp \left[ i (\tau \hat{a} + \tau \bar{\hat{a}}) \right]
= \frac{e^{-\frac{i\tau}{2}}}{\sqrt{2}} \left[ e^{i(\tau \hat{a} + \tau \bar{\hat{a}})} - e^{i(-\tau \hat{a} + \tau \bar{\hat{a}})} \right]
\end{equation}

(a) We see from the expression of \( \psi(\tau, 0) \) that the \( \mathbf{\hat{a}} \) vector has two possible values:
\[ \begin{align*}
\hat{a}_1 &= \hat{a} + \hat{b} - \hat{c} \\
\hat{a}_2 &= -\hat{a} + \hat{b} + \hat{c}
\end{align*} \]
Each has magnitude \( \sqrt{(\hat{a}_1)^2 + (\hat{b}_1)^2 + (\hat{c}_1)^2} = \sqrt{3} \) = 1.73.
Hence, the possible value of energy
\[ E = \frac{k^2 (\sqrt{3})^2}{2m} \]

(b) \[ \begin{align*}
\hat{a}_1 &\rightarrow \text{Prob. of } \frac{1}{2} \\
\hat{a}_2 &\rightarrow \text{Prob. of } \frac{1}{2}
\end{align*} \]

(c) \[ \begin{align*}
\psi(\tau, t) &= \frac{e^{-\frac{i\tau}{2}}}{\sqrt{2}} \left( e^{i\frac{\pi}{4}} m^1 \right) \exp \left[ \frac{i \tau}{\hbar} \left( \hat{a} + \bar{\hat{a}} \right) \right] \exp \left[ \frac{-i \tau}{\hbar} \left( \hat{a} + \bar{\hat{a}} \right) \right]
= \frac{e^{-\frac{i\tau}{2}}}{\sqrt{2}} \left( e^{i\frac{\pi}{4}} m^1 \right) \exp \left[ i \frac{\tau}{\hbar} \left( \hat{a} + \bar{\hat{a}} \right) \right] \exp \left[ \frac{i\tau}{2} \right]
\end{align*} \]

Where
\[ E = \frac{k^2 (\sqrt{3})^2}{2m} \]
Problem 12.10

(i) This is a free particle, and the solution of free particle problem can be obtained from the radial part of the wave function, which is:

\[
\frac{d}{dr} \left( r \cdot \phi' \right) = \frac{2mE}{h^2} \left( \psi(r) - \beta \right) R = \beta (r + 1) R
\]

Setting \( \psi(r) = 0 \), we get:

\[
\frac{d}{dr} \left( r^2 \cdot \phi' \right) + \frac{2mE}{h^2} R = (\beta + 1) \cdot R
\]

The above equation can be further simplified by defining \( \psi(r) = \alpha R(r) \), and \( k = \frac{\sqrt{2mE}}{h} \); the simplified equation would be:

\[
\frac{d}{dr} \left( r^2 \cdot \phi' \right) - k^2 r^2 \psi = (k^2 + 1) \psi
\]

The solution \( \psi \) to this equation is of the form—(spherical Bessel function) \( \times \).
Hence, \( R(r) \) which is \( u(r) \) becomes:

\[
\text{constant}) \; j_l(kr),
\]

where \( j_l \) is the spherical Bessel function of order \( l \). We look in the table and find:

\[
j_l(r) = \frac{\sin kr}{kr} - \frac{\cos kr}{r}
\]

Hence:

\[
j_l(kr) = \frac{\sin kr}{kr} - \frac{\cos kr}{kr}
\]

\( \psi(\vec{r'},0) \) in the problem can be written as:

\[
\psi(\vec{r'},0) = (4\pi i) \; j_1(kr) \; \frac{3y^0(0,0) + 5 y^{-1}(0,0)}{\sqrt{54}}
\]

\[
= (4\pi i) \left[ \frac{3}{\sqrt{54}} j_1(kr) y^0 + \frac{5}{\sqrt{54}} j_1(kr) y^{-1} \right]
\]

Energy depends on \( k \) only:

\[
u = \frac{\sqrt{mE}}{k}, \quad \text{or} \quad E = \frac{\hbar^2 k^2}{2m}
\]

Hence:

\[
\psi(\vec{r'},t) = (4\pi i) \left[ \frac{3}{\sqrt{54}} j_1(kr) y^0 e^{-i\nu kr/\hbar} + \frac{5}{\sqrt{54}} j_1(kr) y^{-1} e^{-i\nu kr/\hbar} \right]
\]

where \( E = \frac{\hbar^2 k^2}{2m} \)

(b) Since, \( \psi(\vec{r'},0) \) is an energy eigenstate (because \( j_1(kr) y^0 \) and \( j_1(kr) y^{-1} \) have the same energy), we can write:

\[
\langle E \rangle = \frac{\hbar^2 k^2}{2m}
\]
Given $l = 2$, the value of $L^2$ would be $h^2 L(l+1) = h^2 (2)(3) = 6h^2$.

The probability of $L^2 = 2h^2$ is $1$. Hence all $N$ neutrons will have this value.

The probability of $L^2 = 0$ is 

\[
\frac{\sqrt{3} \psi_1(0)}{2}\left(\frac{\sqrt{2}}{3} + \frac{\sqrt{3}}{3}\right) = \frac{\sqrt{6}N}{3N} = \frac{\sqrt{6}N}{3N}
\]

The probability of $L^2 = -2h^2$ is 

\[
\frac{-\sqrt{3} \psi_1(0)}{2}\left(\frac{\sqrt{2}}{3} - \frac{\sqrt{3}}{3}\right) = \frac{-\sqrt{6}N}{3N} = \frac{-\sqrt{6}N}{3N}
\]

Hence, $\frac{\sqrt{6}N}{3N}$ neutrons will have value $L^2 = 0$.

$\frac{-\sqrt{6}N}{3N}$ neutrons will have value $L^2 = -2h^2$.

(c) We saw before that there is only one possible value of $L^2$, and that is $2h^2$. $\psi(r,0)$ is an eigenstate of $L^2$, and hence measurement of $L^2$ will leave the system in its eigenstate. Hence $\psi(r,0)$ will be the same as mentioned in part (a).

\[
\psi(r,0) = \left(4\pi i\right) \left[ \frac{3}{\sqrt{5} \psi_1(0)} \right] \psi_1(0) \exp \left[ \frac{iE}{\hbar} + \frac{\sqrt{3} \psi_1(0)}{\sqrt{2}N} \right]
\]

where $E = \frac{h^2 k^2}{2m}$.
\( \psi(r) \) will collapses to:
\[
\left[ \frac{\sin k r}{(kr)} - \frac{\cos k r}{k r} \right] \psi_0
\]

Hence,
\[
\psi(r,t) = \left[ \frac{\sin k r}{(kr)} - \frac{\cos k r}{k r} \right] \psi_0 \quad \text{where} \quad r = \frac{t^2 - x^2}{2m}
\]

\[\text{Prob} \quad 10.28 \]

(a) The constant \( A \) can be calculated from:
\[
\int \psi_0^* (r,0) \psi_0 (r,0) r \sin \theta d\theta dr = 2
\]

(b) We will prove some general facts in order to do this problem. We start with the most general form of \( \psi_0 (r) \):
\[
\psi_0 (r) \text{ can be written as:}
\]
\[
\psi_0 (r) = \sum_{l,m} c_{l,m} \psi_{l,m} (\theta, \phi)
\]

We have used \( \psi_{l,m} (\theta, \phi) \) as basis functions.

\( c_{l,m} \) is given by:
\[
c_{l,m} = \int_0^{2\pi} \int_0^\pi R_{l,m} (\theta, \phi) \psi_0 (r, \theta, \phi) \sin \theta d\theta d\phi
\]

---

---
By using basic postulates of quantum mechanics, we can write:

\[ P_{ij} = \sum_k |c_{ik}c_{jk}|^2 \quad --- (i) \]

This is the probability of measuring \( L_i \) and \( L_j \) simultaneously.

If we measure only \( L_i \), the probability is

\[ P_{i\nu} = \sum_{m=1}^L P_{i\nu m} = \sum_{m=1}^L \left( \sum_k |c_{ik}\bar{c}_{j\nu m}|^2 \right) \quad [\text{using (ii)}] \]

\[ = \sum_k \sum_{m=1}^L |c_{ik}\bar{c}_{j\nu m}|^2 \quad --- (ii) \]

If it is only \( L_2 \) that we wish to measure, the probability of obtaining \( n_\alpha \) is

\[ P_{\alpha 2} = \sum_{m=1}^L P_{\alpha 2 m} \]

\[ = \sum_k \sum_{m=1}^L |c_{k\alpha}\bar{c}_{\alpha m}|^2 \quad [\text{using (iii)}] \]

\[ = \sum_k \sum_{m=1}^L |c_{k\alpha}\bar{c}_{\alpha m}|^2 \quad --- (iii) \]

Next, we can consider \( Y(\tau, \theta, \phi) \) as a function of \( \theta \) and \( \phi \) depending on the parameters \( \tau \):

\[ \psi(\tau, \theta, \phi) = \sum_{\nu\alpha} c_{\nu\alpha}(\tau) \times Y_{\nu\alpha}(\theta, \phi) \quad --- (iv) \]
\[ a_{\nu m}(\tau) = \int_0^{2\pi} \int_0^1 \sin \theta \, d\theta \, d\phi \, y_\nu^m(\theta, \phi) \, \psi(\tau, \theta, \phi) \]  

Comparing (vi) with (i), we see,

\[ a_{\nu m}(\tau) = \sum_k c_{\nu k} m \, R_{\nu k}(\tau) \]  

The next one we will show,

\[ \int_0^{2\pi} d\phi \, |a_{\nu m}(\tau)|^2 = \sum_k |c_{\nu k} m|^2 \]  

**Proof:**  
Using (viii), we can write -

\[ |a_{\nu m}(\tau)|^2 = a_{\nu m}(\tau) \, a^{*}_{\nu m}(\tau) \]

\[ = \sum_k \sum_k c_{\nu k} m \, c_{\nu k}^* m \, R_{\nu k}(\tau) \, R^{*}_{\nu k}(\tau) \]

then,

\[ \int_0^{2\pi} d\phi \, |a_{\nu m}(\tau)|^2 = \sum_k |c_{\nu k} m|^2 \int_0^{2\pi} d\phi \int_0^1 \sin \theta \, d\theta \, d\phi \]

\[ = \sum_k |c_{\nu k} m|^2 \, \left[ \int_0^{2\pi} R_{\nu k}(\phi) \, R^{*}_{\nu k}(\phi) \, d\phi \right] \]

\[ = \sum_k |c_{\nu k} m|^2 \]

\[ \text{END OF PROOF} \]

Next, we will refer to (iii), and can write
referring to (iv) and (ix) we can write

\[ P_{v2} = \frac{\phi L}{2\pi} \int_0^\infty r e^{-r} |a_{lm}(r)|^2 dr \]  

\[ P_{-v2} = \frac{\phi L}{2\pi} \int_0^\infty r e^{-r} |a_{lm}(r)|^2 dr \]  

\[ \text{referring to (v)} \]  

we will be using (ix) for part (iv).

We compare the expression \( \psi(r, \theta, \phi) \) given in the problem with \( \psi(\rho, \theta, \phi) \) in (vi).

we see:

\[ a_{l=0, m=0}(r) = \frac{1}{(2\pi)^{\frac{3}{2}}} e^{-r/\rho_0} \]

\[ a_{l=1, m=1}(r) = \left( \frac{2}{(2\pi)^{\frac{3}{2}}} \right) e^{-r/\rho_0} \]

\[ a_{l=1, m=-1}(r) = \left( \frac{2}{(2\pi)^{\frac{3}{2}}} \right) e^{-r/\rho_0} \]

\[ a_{l=2, m=2}(r) = \left( \frac{2}{(2\pi)^{\frac{3}{2}}} \right) e^{-r/\rho_0} \]
hence, using (91), the probability that measurement of \( Z \) finds the value \( \theta = \theta_0 \) is

\[
\int_{\theta_0}^{\theta_0} \frac{e^{-r^2}dr}{(2\pi\beta)^{1/2}}.
\]

And the probability that measurement of \( Z \) finds the value

\[
(1) \quad (1 + 1) \beta^2 = 2\beta^2
\]

\[
\int_{0}^{\infty} e^{-r^2} dr |a_{11}(r)|^2 + \int_{0}^{\infty} e^{-r^2} dr |a_{12}(r)|^2
\]

\[
+ \int_{0}^{\infty} e^{-r^2} dr |a_{21}(r)|^2 + \int_{0}^{\infty} e^{-r^2} dr |a_{22}(r)|^2
\]

\[
= \int_{0}^{\infty} e^{-r^2} dr \frac{\beta^2}{\sigma^2} \left( e^{-\frac{\beta^2}{\sigma^2}} \right)^2 \left( 1 + 1 \left( \frac{\sigma}{\beta} \right)^2 \right) \left( \frac{\beta^2}{\sigma^2} \right)^2
\]

\[
= \int_{0}^{\infty} e^{-r^2} dr \frac{\beta^2}{\sigma^2} \left( e^{-\frac{\beta^2}{\sigma^2}} \right)^2 \left( 1 + 1 \left( \frac{\sigma}{\beta} \right)^2 \right) \left( \frac{\beta^2}{\sigma^2} \right)^2
\]

\[
= \frac{\beta^2}{\sigma^2} \left( e^{-\frac{\beta^2}{\sigma^2}} \right)^2 \left( 1 + 1 \left( \frac{\sigma}{\beta} \right)^2 \right) \left( \frac{\beta^2}{\sigma^2} \right)^2
\]

We need to use the value of \( \theta_0 \), the one we get from the normal factor.

(3) The probability density \( p(\theta) \) is obtained as

\[
\int_{\theta_0}^{\theta_0} \frac{e^{-r^2}dr}{(2\pi\beta)^{1/2}} \text{ for } \theta = \theta_0,
\]

\[
\int_{0}^{\infty} \left[ e^{-r^2} dr \right] \text{ for } \theta_0 < \theta < \infty.
\]

(6, and \( \theta \) are integrating out.)
(b) $P_2(r)$ is maximum at the value of $r$ for which $\frac{dP_2}{dr} = 0$.

(c) \[ \hbar = n \text{ in the notations we are using, it is } \hbar, \text{ but it is also used as } n \text{ in books} \]

In our problem, we see, \( n \) has two possible values:

\[ n = 1, \quad n = 2 \]

We refer to the table in page 455 for \( \hbar \).

Hence, possible energies are:

\[ E_1 = -13.6 \text{ eV} \quad \text{and} \quad E_2 = -13.6 \text{ eV} \]

\[ \psi(r, \theta, \phi) = \frac{\Psi_1}{\sqrt{2\pi a_0}} \frac{e^{iE_1 \phi/\hbar}}{r \sqrt{2 \pi a_0}} \overline{\chi_0} \chi_1 \[ \frac{1}{\sqrt{2\pi a_0}} \frac{e^{iE_2 \phi/\hbar}}{r \sqrt{2 \pi a_0}} \overline{\chi_0} \chi_1 \]

\[ + \frac{1}{\sqrt{2\pi a_0}} \frac{e^{iE_2 \phi/\hbar}}{r \sqrt{2 \pi a_0}} \overline{\chi_0} \chi_1 \]

\[ \Psi_e(r, \theta, \phi) = \Psi_1 \Psi_e \overline{\chi_0} \chi_1 \]

The constants \( \Psi_1 \Psi_2 \overline{\chi_0} \chi_1 \)

are equal to the multiplicity of the states.

\[ E_1 \text{ and } E_2 \text{ have the values } -13.6 \text{ eV} \quad \text{and} \quad -13.6 \text{ eV} \text{ respectively.} \]
(f) $\psi(r^2, 0)$ will become:

$$\psi(r^2, 0) = \frac{4A}{(2\pi \hbar)^{3/2}} e^{-r^2/2\alpha^2} (-i) Y_1^1$$

hence,

$$\psi(r^2, t) = \frac{4A}{(2\pi \hbar)^{3/2}} e^{-r^2/2\alpha^2} (-i) Y_1^1 e^{-iE_2 t/\hbar}$$

where $E_2 = -13.6 \text{eV}$

(g) If measurement of $x_2$ is $0^\circ$ —

$$\psi(x^2, 0)$$ will collapse to:

$$\frac{\psi_{x_2}}{(2\pi \hbar)^{3/2}} e^{-r^2/2\alpha^2} \left[ Y_0^0 = \frac{1}{\sqrt{4\pi}} \right]$$

hence,

$$\psi(x^2, t) = \frac{4A}{(2\pi \hbar)^{3/2}} e^{-r^2/2\alpha^2} \left[ Y_0^0 = \frac{1}{\sqrt{4\pi}} \right]$$

where $E_1 = -13.6 \text{eV}$

$[E2$ because $e^{-r^2/2\alpha^2}$ e-folding to $n = 2^+]$

(h) From $\theta < 45^\circ$ half off,

$$\theta = \frac{\hbar}{2n} = \frac{\hbar}{2\pi a}$$
\[ \langle \psi(r, 0) \mid \hat{H} \mid \psi(r, 0) \rangle = \langle \psi(r, 0) \mid \frac{\hat{L}_z^2}{2\hbar^2} + \psi(r, 0) \rangle \quad - (2) \]

\( \psi(r, 0) \) is written in terms of spherical harmonics.

\[ \psi(r, 0) = \frac{1}{(2\pi\hbar)^{3/2}} \int e^{-i\lambda_0 \hat{r}} \left( \sum_{l=0}^{\infty} c_l e^{-i\lambda_l \hat{r}} \right) \psi_l \]

\[ \text{below,} \]

\[ \mathcal{C}_l \psi(r, 0) = \frac{1}{(2\pi\hbar)^{3/2}} \int e^{i\lambda_0 \hat{r}} \left( \sum_{l=0}^{\infty} c_l e^{i\lambda_l \hat{r}} \right) \psi_l \]

\[ \text{We have used the} \]

\[ \text{fact} \quad \sum_{l=0}^{\infty} c_l e^{-i\lambda_l \hat{r}} = 2\pi \delta \left( r - r_0 \right) \]

\[ \text{in 3-dimension expectation is defined as} \]

\[ \langle \psi(r, 0) \mid \frac{\hat{L}_z^2}{2\hbar^2} + \psi(r, 0) \rangle = \int \int \psi^*(r, 0) \left( \frac{\hat{L}_z^2}{2\hbar^2} \right) \psi(r, 0) \]
As mentioned before, there are two possible energy values:

\[ E_1 = -13.6 \text{ eV} \quad \text{and} \quad E_2 = -13.6 \text{ eV} \]

Hence, the lowest value is \( E_1 = -13.6 \text{ eV} \).