2 LRC Circuit

Transient Response

Purpose:

To investigate the transient effects in a circuit containing and inductance (L), a resistance (R), and a capacitance (C). To gain an understanding of the parameters that produce three separate transient (damping) conditions, known as the underdamped, overdamped and critically damped conditions.

Background :

The equation representing the series LRC circuit with a constant applied voltage, $V_{0,}$ is:

$$L\left(\frac{d^2q}{dt^2}\right) + R\left(\frac{dq}{dt}\right) + \left(\frac{q}{C}\right) = V_0 \qquad \textbf{3.1}$$

Making the substitution $Q = q - CV_{0,}$ the equation becomes:

$$L\left(\frac{d^2Q}{dt^2}\right) + R\left(\frac{dQ}{dt}\right) + \left(\frac{Q}{C}\right) = 0 \qquad \textbf{3.2}$$

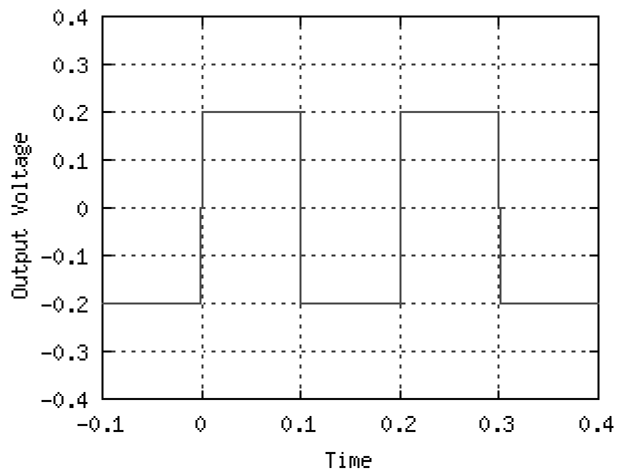
This equation could also be written:,

$$m\left(\frac{d^2x}{dt^2}\right) + b\left(\frac{dx}{dt}\right) + (kx) = 0$$

where we have substituted x for Q, m for L, b for R and k for 1/C. This is an equation for a simple harmonic oscillator with a frictional damping force proportional to the velocity. So we may expect this LRC circuit to have behavior similar to a damped simple harmonic oscillator.

Because the oscillation decreases with time, and eventually disappears, the solution of Equation 3.2 is a “transient” solution. Since all terms containing the dependent variable (Q) are to the first power, the equation is “linear.” Since the highest order of the derivative of the dependent variable is two, the equation is “second-order.” Since the right hand side of Equation 3.2 is identically zero, we have a so-called “homogenous” equation. A derivation of the solution is given in the Appendix. In this laboratory, the effect of different values of the parameters L, R , and C will be studied experimentally.

In this experiment, we use a signal generator with a rectangular output waveform, an example of which is shown here:



At time t = 0, the voltage has been at $-V_0$ for a "long" time; any current from the previous transition has decayed to zero, and the voltage on the capacitor is $-V_0$. With these two initial conditions, the charge can be shown to be:

$$q = CV_0 - 2CV_0 e^{(-\alpha t)}\left[cos(w_1 t) + \frac{\alpha}{w_1} sin(w_1 t)\right] \qquad \textbf{3.3}$$

where q is the instantaneous charge, t is time, and w_1 is the "angular frequency" of the circuit with total resistance R and

$$\alpha = R/2L \qquad \textbf{3.4}$$

The second term of the equation is sometimes very small.
The angular frequency of the circuit with resistance (w) is sometimes called the reduced frequency because:

$$w = \left[\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2\right]^{1/2} \qquad \textbf{3.5}$$

Recall that the "natural" frequency of a LC circuit without resistance is:

$$w_o = \left(\frac{1}{LC}\right)^{1/2} \qquad \textbf{3.6}$$

Therefore,

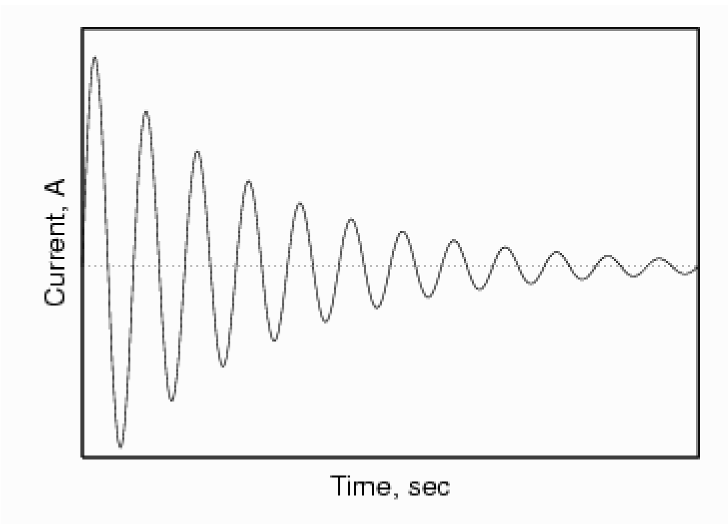
$$w = [(w_o)^2 - (\alpha)^2]^{1/2} \qquad \textbf{3.7}$$

so that in order to have oscillations, $\alpha < w_o$, and

$$w < w_o$$

Examination of Equation 3.2 indicates that the solution of Equation 3.1 is a sinusoidal waveform, with angular frequency w, having an initial amplitude $2CV_0$, which decays exponentially. The frequency w and the rate of decay are determined by the LRC parameters of the circuit.

There are three ranges to be investigated experimentally:



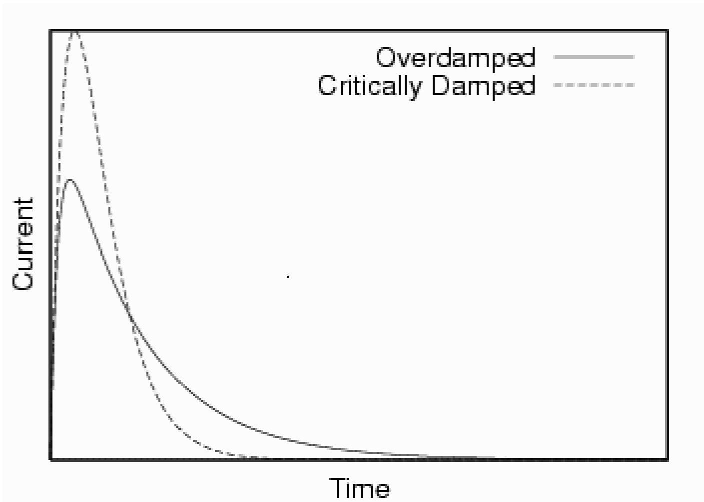
$\alpha = (R/2L) < w_o$ (damped oscillations)

$(R/2L) > w_o$ (no oscillation due to overdamping)

$(R/2L) = w_o$ (no oscillation, minimum time to zero, this is critical damping)

When $(R/2L) < w_o$, the circuit is damped and will oscillate with decreasing amplitude. If (R/2L) is sufficiently small w can approach w_o and the amplitude can decay very slowly. Graphically, this condition is illustrated in the figure above.

When $(R/2L) > w_o$, the circuit is overdamped. It does not oscillate at all. In fact, it can take an excessive amount of time for the charge to decrease to zero, i.e., for the circuit to return to equilibrium. Graphically this condition is illustrated in the figure below.
When $(R/2L) = w_o$, the circuit is critically damped. Again, it does not oscillate at all, hence it is similar in appearance to the overdamped condition. However, the charge decreases to zero in the most expeditious manner, i.e., the circuit returns to equilibrium as quickly as possible without oscillation. Graphically, it is similar to the overdamped case. However, for critical damping, the time will be shorter than for overdamping. Critical damping is compared to overdamping in the figure below



It is also possible to show that the current in the circuit is

$$i = 2CV_0 e^{(-\alpha t)}\left(\frac{\alpha^2 + \omega^2}{\omega}\right) sin(wt)$$

and the voltage across the resistor is just -iR.

Experimental Setup

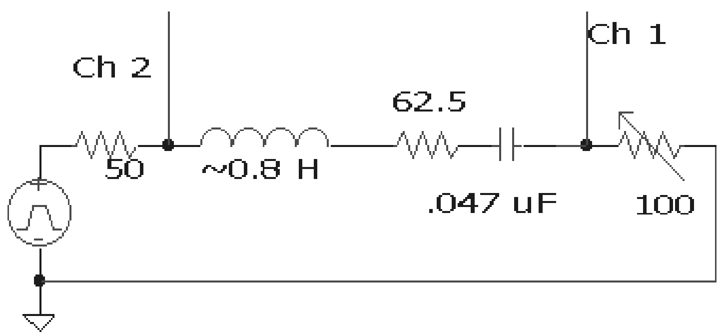


Figure 3.1: Setup for measuring V_R in the LRC circuit. The 50 and 62.5 ohm resistors are the internal resistances of the generator and the inductor

Procedures:

Set up the circuit shown in Figure 3.1. The square wave generator has the same effect as a battery or power supply being switched on and off. Figure 3.1: shows the oscilloscope measuring the voltage across the resistor. In calculating the decay constant α the resistance R is the total resistance in the circuit, including the resistance of the signal generator, as well as the variable load resistor ("pot") and the inductor. But the voltage drop across the load resistor is just iR_{load}.

Calculate the angular frequency w_o and the corresponding frequency f_o. The signal generator must have a frequency considerably less than f_o, at least a factor of 10 times lower. Explain why.

Underdamped Circuit: Set the signal generator square wave amplitude to about 5.0V. Measure and record its value. Trigger the oscilloscope on Channel 2 (the signal generator output) and look at Channel 1. Adjust the timing and vertical gain to fill the display with 6-10 cycles of the oscillation. Then read out the scope with the Wave Star program. Setting the tabular display with the time included, copy the file to the clipboard, and then paste it into a spreadsheet. In the spreadsheet compute the predicted V_R , and make a plot which shows data and prediction together on the same graph. It is a good idea to at least a rough check for agreement between data and prediction before you leave.

For the underdamped circuit data, compute the rms deviation of the data from the prediction. You will need to restrict the times to positive values to make this meaningful. Then make small changes in the “parameters” R, C, L and V_0 in order to minimize the rms deviation. It is best to be looking at the graph while making these changes. Give the results in your report.

Critically Damped Circuit: increase the resistance of the variable resistor until critical damping is reached, i.e. just between oscillation and overshooting. Check to see if $(R/2L) = w_o = (1/LC)^{1/2}$. Remember the resistance of the pot should be measured with the signal generator off or with the pot out of the circuit. Also, you must use the total resistance of the circuit that includes the signal generator and the inductor.

Overdamped Circuit: continue increasing the resistance. Describe pictorially what happens.