Reflection of Light from a Dielectric: Fresnel Equations

1 Purpose

The intensity of light reflected from the surface of a plexiglas block is measured as function of the angle of incidence. This data is compared with the predictions of electromagnetic theory, given by the Fresnel equations\(^1\). While this subject is generally not covered in an introductory physics textbook, the phenomenon is widely observed, and suitable for an experiment.

2 Introduction

When an electromagnetic wave strikes the surface of a dielectric, both reflected and refracted waves are generally produced. The reflected wave has a direction given by the “Law of Reflection”:

\[ \theta_r = \theta_i \]

where the angles are between the rays and a line perpendicular to the reflecting surface.

Electromagnetic theory predicts the ratio of the intensity of the reflected light to the intensity of the incident light. The polarization of the light with respect to the plane of reflection \(^2\) must be taken into account. There are two extreme cases: (1) the electric field is perpendicular to the plane of reflection, called Transverse Electric, and (2) the magnetic field is perpendicular to the plane of reflection, Transverse Magnetic.

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\(^1\)Deduced by Augustin Fresnel in 1827, many decades before the formal development of electrodynamics by Maxwell.

\(^2\)The plane of reflection is the plane defined by the incident and reflected rays.
These two cases are illustrated in the figure below:

Figure 1: TE and TM reflections. The dot shows the direction of the electric field in the TE case, and the magnetic field in the TM case.

For the transverse electric case, it can be shown that the intensity reflection coefficient,

\[ R_{TE} = \frac{I_r}{I_i} = \left( \frac{\cos \theta - \sqrt{n^2 - \sin^2 \theta}}{\cos \theta + \sqrt{n^2 - \sin^2 \theta}} \right)^2 \]  

in which \( \theta = \theta_i = \theta_r \), and \( n = n_2/n_1 \). For the transverse magnetic case, the result is

\[ R_{TM} = \frac{I_r}{I_i} = \left( \frac{-n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}}{n^2 \cos \theta + \sqrt{n^2 - \sin^2 \theta}} \right)^2 \]  

The intensity reflection coefficients for TE and TM cases are plotted vs incident angle in Figure 2, for \( n = 1.5 \).

Figure 2: Intensity reflection coefficients for \( n = 1.5 \)
Note that $R_{TM} = 0$ at the Brewster angle, which is $56.3^\circ$ for $n = 1.5$.

3 Apparatus

The apparatus comprises a red laser diode, a polarizing filter, a semi-cylindrical plexiglas block on a rotating table, and a light sensor. The light sensor is voltage-biased photodiode. The output is a voltage which is proportional to the light intensity incident on the photodiode. The sensor requires +10 volts to bias the diode. A photograph of the apparatus is shown in Figure 3.

Figure 3: Apparatus

The figure below shows the laser beam being reflected by the semi-cylinder into the photodiode

Figure 4: Diagram of reflection
4 Procedure

1. Set up the apparatus; connect a voltmeter and voltage supply to the light sensor. Set the supply to +10 volts.

2. Set the polaroid filter to 45°. This is done by rotating the filter until the 45° mark is opposite the dot on the bottom of the filter. This makes the angle of the \( E \)-vector from the vertical equal to 45°. Let’s call this angle \( \phi_{pol} \).

3. Adjust the rotating table to approximately the angle desired. Then rotate the semi-cylinder block so that the laser beam is reflected into the photodiode. Use the adjustment screws on the back of the laser diode to set the beam to hit the center of the photodiode.

4. With a piece of paper or an index card, measure the angle reading on the rotating table for the incident and reflected beams. Hold the card so that an edge is perpendicular to the table and move it until you can just see the beam hitting the edge of the card. Then read the angle at the location of the bottom of the card. From these two angles find the quantity \( 2\theta \) and from that \( \theta \).

5. Measure the photo-diode output at this angle. Then cover up the laser beam, at the laser, and read the output again. This will give the voltage due to background light. Subtract the background voltage. Note this must be done at each angle, since the background light will change with angle.

6. Make at least 5 measurements within the range 0° to 90°.

7. Then remove the semi-cylinder, and measure the photo-diode voltage when the laser beam goes directly into it, with no reflection. This voltage, \( V_{head-on} \), is used to compare the data with predictions.

5 Data Analysis

1. Enter the data into a spread sheet.

2. The predicted voltages are obtained from the equation

   \[
   V_{pred} = V_{head-on}(R_{TE} \cos^2 \phi_{pol} + R_{TM} \sin^2 \phi_{pol})
   \]

   where \( R_{TE} \) and \( R_{TM} \) are obtained from Equations 1 and 2. Use \( n = 1.50 \) for plexiglas.

3. Make a plot which shows \( V_{pred} \), \( V_{TE} = V_{head-on}R_{TE} \), \( V_{TM} = V_{head-on}R_{TM} \), and the data points, as functions of the angle of incidence.