Heat Engines

Lecture #6
HNRS 228
Energy and the Environment

Heat Engines, Heat Pumps, and Refrigerators
Object: Getting something useful from heat

Heat can be useful
- Normally heat is the end-product of the flow/transformation of energy
  - Consider examples coffee mug, automobile, bouncing ball
  - Typically heat regarded as waste
  - Useless end result
- Sometimes heat is what we want
  - e.g. hot water, cooking, space heating
  - Heat can also be coerced into performing "useful" (e.g., mechanical) work
  - This is called a "heat engine"

Heat Engine Concept
- If a temperature difference exists between two bodies
  - Then there is a potential for heat flow
- Examples:
  - Heat flows out of a hot pot of soup
  - Heat flows into a cold drink
  - Heat flows from the hot sand into your feet
- Rate of heat flow depends on
  - Nature of contact
  - Thermal conductivity of materials
- Some of this flow of energy can be transformed into mechanical work

Heat → Work
- Examples of heat energy transformed into other types of energy
  - Air over a hot car roof is lofted
    - Gains kinetic energy
    - Also gains gravitational potential energy
  - Wind is driven by temperature differences
  - Think about radiative heat energy transfer
  - Electricity generation thrives on temperature differences
    - No steam would circulate if everything was at the same temperature

Power Plant Arrangement

Figure 2: A diagram of a hypothetical electric power plant. Hot air provides cooling water to the condenser. Hot water in a cooling tower could serve the same purpose.

Heat flows from $T_h$ to $T_c$, turning turbine along the way
Heat Engine Nomenclature
- The symbols used to describe a heat engine are:
  - $T_h$ is the temperature of the hot object (typ. in Kelvin)
  - $T_c$ is the temperature of the cold object (typ. in Kelvin)
  - $\Delta T = T_h - T_c$ is the temperature difference
  - $\Delta Q_h$ is the amount of heat that flows out of the hot body
  - $\Delta Q_c$ is the amount of heat flowing into the cold body
  - $\Delta W$ is the amount of "useful" mechanical work
  - $\Delta S_h$ is the change in entropy of the hot body
  - $\Delta S_c$ is the change in entropy of the cold body
  - $\Delta S_{tot}$ is the total change in entropy (entire system)
  - $\Delta E$ is the entire amount of energy involved in the flow

What's this Entropy business?
- Entropy is a measure of disorder (and actually quantifiable on an atom-by-atom basis)
- Ice has low entropy, liquid water has more, steam has much more

What is the generic name for a cyclical device that transforms heat energy into work.
A. Refrigerators
B. Thermal Motors
C. Heat Engines
D. Carnot Cycles
E. Otto processors

The Laws of Thermodynamics
1. Energy is conserved
2. Total system entropy can never decrease
3. As the temperature goes to zero, the entropy approaches a constant value—this value is zero for a perfect crystal lattice
- The concept of the "total system" is very important: entropy can decrease locally, but it must increase elsewhere by at least as much
- no energy flows into or out of the "total system": if it does, there's more to the system than you thought

Quantifying heat energy
- Quantifying heat
  - 1 Calorie is the heat energy associated with raising 1 kg (1 liter) of water 1 °C
  - In general, $\Delta Q = c_p \Delta T$, where $c_p$ is the heat capacity
  - A change in heat energy accompanies a change in entropy:
    - $\Delta Q = T \Delta S$
    (T expressed in K)
- Adding heat increases entropy
  - more energy goes into random motions—more randomness (entropy)
How much work can be extracted from heat?

Hot source of energy

\[ \Delta Q_h \]

heat energy delivered from source

externally delivered work:

\[ \Delta W = \Delta Q_h - \Delta Q_c \]

heat energy delivered to sink

Cold sink of energy

\[ \Delta Q_c \]

efficiency = \[ \frac{\Delta W}{\Delta Q_h} \] = \[ \frac{\text{work done}}{\text{heat supplied}} \]

Let's crank up the efficiency

Let's extract a lot of work, and deliver very little heat to the sink.

In fact, let's demand 100% efficiency by sending no heat to the sink: all converted to useful work.

\[ \Delta W = \Delta Q_h - \Delta Q_c \]

It's a really hot day and your air conditioner is broken.

Your roommate says, “Let’s open the refrigerator door and cool this place off.” Will this work?

A. Yes.
B. It might, but it will depend on how hot the room is.
C. No.

Rank in order, from largest to smallest, the work \( W_{\text{net}} \) performed by these four heat engines.

A. \( W_a > W_b > W_c > W_d \)
B. \( W_a > W_b > W_c = W_d \)
C. \( W_a > W_b = W_c > W_d \)
D. \( W_a = W_b > W_c > W_d \)
E. \( W_d > W_c > W_b > W_a \)
Not so fast...

- The second law of thermodynamics imposes a constraint on this reckless attitude: total entropy must never decrease.
- The entropy of the source goes down (heat extracted), and the entropy of the sink goes up (heat added); remember that $\Delta Q = T \Delta S$.
- The gain in entropy in the sink must at least balance the loss of entropy in the source:
  $$\Delta S_{\text{sink}} = \Delta S_h + \Delta S_c = -\frac{\Delta Q_h}{T_h} + \frac{\Delta Q_c}{T_c} \geq 0$$
  $\Delta Q_c \geq (T_c/T_h) \Delta Q_h$ sets a minimum on $\Delta Q_c$.

What does this entropy limit mean?

- $\Delta W = \Delta Q_h - \Delta Q_c$, so $\Delta W$ can only be as big as the minimum $\Delta Q_h$ will allow.
- $\Delta W_{\text{max}} = \Delta Q_h = \Delta Q_h, \Delta Q_c = \Delta Q_h (T / T_h)$.
- So the maximum efficiency is:
  $$\text{max efficiency} = \frac{\Delta W_{\text{max}}}{\Delta Q_h} = \frac{T - T_c}{T_h} \frac{\Delta Q_h}{T_h}$$
  this and similar formulas must have the temperature in Kelvin.
- So perfect efficiency is only possible if $T_c$ is zero (in °K).
- In general, this is not true.
- As $T_c \to T_h$, the efficiency drops to zero: no work can be extracted.

Examples of Maximum Efficiency

- A coal fire burning at 825 °K delivers heat energy to a reservoir at 300 °K.
  - max efficiency is $(825 - 300)/825 = 525/825 = 64\%$.
  - this power station can not possibly achieve a higher efficiency based on these temperatures.
- A car engine running at 400 °K delivers heat energy to the ambient 290 °K air.
  - max efficiency is $(400 - 290)/400 = 110/400 = 27.5\%$.
  - not too far from reality.

What, if anything, is wrong with this refrigerator?

- A. It violates the first law of thermodynamics.
- B. It violates the second law of thermodynamics.
- C. It violates the third law of thermodynamics.
- D. It violates both the first and second law of thermodynamics.
- E. Nothing is wrong.

Could this heat engine be built?

- A. Yes.
- B. No.
- C. It’s impossible to tell without knowing what kind of cycle it uses.

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What to do with the waste heat ($\Delta Q_c$)?

- One option: use it for space-heating locally

Overall efficiency greatly enhanced by cogeneration

Table 3.1 Cogeneration Plant, University of Colorado, Boulder

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Natural gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td>2 Mitsubishi industrial gas turbines</td>
</tr>
<tr>
<td>Generating capacity</td>
<td>32 MW</td>
</tr>
<tr>
<td>Capital investment</td>
<td>$45,000,000</td>
</tr>
<tr>
<td>Construction started</td>
<td>1990</td>
</tr>
<tr>
<td>System lifetime</td>
<td>40 to 50 years</td>
</tr>
<tr>
<td>Estimated payback time</td>
<td>15 years</td>
</tr>
<tr>
<td>Average exported electric power</td>
<td>8 MW</td>
</tr>
<tr>
<td>Cost of electricity produced</td>
<td>$0.0545/kWh</td>
</tr>
<tr>
<td>Price of electricity sold</td>
<td>$0.087/kWh</td>
</tr>
<tr>
<td>Annual income from electricity sales</td>
<td>$1,600,000</td>
</tr>
<tr>
<td>Cost of electricity from public utility</td>
<td>$0.0683/kWh</td>
</tr>
<tr>
<td>Efficiency for producing electricity</td>
<td>34%</td>
</tr>
<tr>
<td>Overall efficiency</td>
<td>70%</td>
</tr>
</tbody>
</table>

Heat Pumps provide a means to efficiently move heat around, and work both in the winter and the summer.
Heat Pumps and Refrigerators: Thermodynamics

\[ \Delta Q_h \Delta Q_c \Delta W = \Delta Q_h - \Delta Q_c \]

- Hot entity (indoor air)
- Cold entity (outside air or refrigerator)
- Just a heat engine run backwards...

- Heat energy delivered
- Heat energy extracted
- Delivered work:
  \[ \Delta W = \Delta Q_h - \Delta Q_c \]
- Conservation of energy: Just a heat engine run backwards...

Efficiency = \[ \frac{\Delta W}{\Delta Q_h} \] (heat pump)
Efficiency = \[ \frac{\Delta W}{\Delta Q_c} \] (refrigerator)

Heat Pump/Refrigerator Efficiencies

- Work through similar logic as before to see:
  - Heat pump efficiency is: \[ T_h (T_h - T_c) = T_h / \Delta T \] in °K
  - Refrigerator efficiency is: \[ T_c (T_h - T_c) = T_c / \Delta T \] in °K
- Note that heat pumps and refrigerators are most efficient for small temperature differences:
  - Hard on heat pumps in very cold climates
  - Hard on refrigerators in hot settings

Example Efficiencies

- A heat pump maintaining 20 °C when it is -5 °C outside has a maximum possible efficiency of:
  \[ \frac{293}{25} = 11.72 \]
  - Note that this means you can get almost 12 times the heat energy than you are supplying in the form of work!
  - This factor is called the C.O.P. (Coefficient of Performance)
- A freezer maintaining -5 °C in a 20 °C room has a maximum possible efficiency of:
  \[ \frac{268}{25} = 10.72 \]
  - Called EER (Energy Efficiency Ratio)

Example Labels (U.S. & Canada)

Again - First Law of Thermodynamics

- The First Law of Thermodynamics tells us that the internal energy of a system can be increased by:
  - Adding energy to the system
  - Doing work on the system
- There are many processes through which these could be accomplished
  - As long as energy is conserved

Again - Second Law of Thermodynamics

- Constrains the First Law
- Establishes which processes actually occur
- Heat engines are an important application
Work in Thermodynamic Processes - Assumptions

- Dealing with a gas
- Assumed to be in thermodynamic equilibrium
  - Every part of the gas is at the same temperature
  - Every part of the gas is at the same pressure
- Ideal gas law applies

Work in a Gas Cylinder

- A force is applied to slowly compress the gas
  - The compression is slow enough for all the system to remain essentially in thermal equilibrium
  - \( W = -P \Delta V \)
  - This is the work done on the gas

More about Work on a Gas Cylinder

- When the gas is compressed
  - \( \Delta V \) is negative
  - The work done on the gas is positive
- When the gas is allowed to expand
  - \( \Delta V \) is positive
  - The work done on the gas is negative
- When the volume remains constant
  - No work is done on the gas

Notes about the Work Equation

- If pressure remains constant during the expansion or compression, this is called an isobaric process
- If the pressure changes, the average pressure may be used to estimate the work done

PV Diagrams

- Used when the pressure and volume are known at each step of the process
- The work done on a gas that takes it from some initial state to some final state is the negative of the area under the curve on the PV diagram
- This is true whether or not the pressure stays constant

More PV Diagrams

- The curve on the diagram is called the path taken between the initial and final states
- The work done depends on the particular path
- Same initial and final states, but different amounts of work are done
iClicker Question
- By visual inspection, order the PV diagrams shown below from the most negative work done on the system to the most positive work done on the system.
- Hint: Use area formulae for triangles and rectangles.
  a) a,b,c,d  b) a,c,b,d  c) d,b,c,a  d) d,a,c,b

Carnot Engine
- A theoretical engine developed by Sadi Carnot
- A heat engine operating in an ideal, reversible cycle (now called a Carnot Cycle) between two reservoirs is the most efficient engine possible
- Carnot's Theorem: No real engine operating between two energy reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs

Carnot Cycle
- A to B is an isothermal expansion at temperature $T_h$.
  - The gas is placed in contact with the high temperature reservoir.
  - The gas absorbs heat $Q_h$.
  - The gas does work $W_{AB}$ in raising the piston.

Carnot Cycle, A to B
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  - The gas does work $W_{AB}$ in raising the piston.

Carnot Cycle, B to C
- B to C is an adiabatic expansion.
  - The base of the cylinder is replaced by a thermally nonconducting wall.
  - No heat enters or leaves the system.
  - The temperature falls from $T_h$ to $T_c$.
  - The gas does work $W_{BC}$.

Carnot Cycle, C to D
- The gas is placed in contact with the cold temperature reservoir at temperature $T_c$.
  - C to D is an isothermal compression.
  - The gas expels energy $Q_C$.
  - Work $W_{CD}$ is done on the gas.
Carnot Cycle, D to A

- D to A is an adiabatic compression
- The gas is again placed against a thermally non-conducting wall
- So no heat is exchanged with the surroundings
- The temperature of the gas increases from $T_c$ to $T_h$
- The work done on the gas is $W_{CD}$

Carnot Cycle, PV Diagram

- The work done by the engine is shown by the area enclosed by the curve
- The net work is equal to $Q_h - Q_c$

Efficiency of a Carnot Engine

- Carnot showed that the efficiency of the engine depends on the temperatures of the reservoirs
  \[ e_c = 1 - \frac{T_c}{T_h} \]
- Temperatures must be in Kelvins
- All Carnot engines operating between the same two temperatures will have the same efficiency

Notes About Carnot Efficiency

- Efficiency is 0 if $T_h = T_c$
- Efficiency is 100% only if $T_c = 0$ K
- Such reservoirs are not available
- The efficiency increases as $T_c$ is lowered and as $T_h$ is raised
- In most practical cases, $T_c$ is near room temperature, 300 K
  - So generally $T_h$ is raised to increase efficiency

The area enclosed within a $pV$ curve is

A. the work done by the system during one complete cycle.
B. the work done on the system during one complete cycle.
C. the thermal energy change of the system during one complete cycle.
D. the heat transferred out of the system during one complete cycle.
The maximum possible efficiency of a heat engine is determined by

A. its design.
B. the amount of heat that flows.
C. the maximum and minimum pressure.
D. the compression ratio.
E. the maximum and minimum temperature.

The engine with the largest possible efficiency uses a

A. Brayton cycle.
B. Joule cycle.
C. Carnot cycle.
D. Otto cycle.
E. Diesel cycle.

What is the thermal efficiency of this heat engine?

1. 0.10
2. 0.25
3. 0.50
4. 4
5. Can't tell without knowing $Q_C$. 

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