Black Holes
A Matter of Gravity
Guiding Questions

1. What are the two central ideas behind Einstein’s special theory of relativity?
2. How do astronomers search for black holes?
3. What are super massive black holes, and where are they found?
4. In what sense is a black hole “black”?
5. In what way are black holes actually simpler than any other objects in astronomy?
6. What happens to an object that falls into a black hole?
7. Why do some pulsars emit fantastic amounts of X rays?
8. Do black holes last forever?
Good to Know
Introduction to Special Relativity

• Motivation
  – Michelson-Morley Experiment
  – Induction versus Force Law

• The Basics
  – Events
  – Principles of Relativity
  – Giving up on absolute space and time

• What Follows from the Basics
  – Time Dilation
  – Length Contraction
  – Twin Paradox?

• The Big Picture
  – Spacetime
  – Kinematics
The Speed of Light

• Special Relativity becomes important in systems which are moving on the order of the speed of light

• The speed of light is $c=3\times10^8$ m/s is very fast:
  – Is exactly 299,792,458 m/s (how can they know this is the exact speed?)
  – 1 foot per nanosecond
  – 1 million times the speed of sound.
  – Around the earth 7 times in a second
  – Earth to sun in 15 min.

• Galileo was the first person to propose that the speed of light be measured with a lantern relay. His experiment was tried shortly after his death.

• In 1676 Ole Roemer first determined the speed of light (how can this be done with 17’th cent equipment.)
The Speed of Light

- In 1873, Maxwell first understood that light was an electromagnetic wave.
- It was the understanding of the nature of EM radiation which first led to a conceptual problem that required relativity as a solution.
- According to his equations, a pulse of light emitted from a source at rest would spread out at velocity c in all directions.
- But what would happen if the pulse was emitted from a source that was moving?
- This possibility confused physicists until 1905.
In Water Things Look Like This

- A boat moving through water will see forward going waves as going slow and backwards going waves as going fast.
• Albert Michelson and Edward Morley were two American physicists working at Case Western Reserve University in Cleveland.

• They constructed a device which compared the velocity of light traveling in different directions (1887).

• They found, much to their surprise that the speed of light was identical in all directions!

• This is strange?????
• If the aether theory were correct, light would thus move more slowly against the aether wind and more quickly downwind. The Michelson-Morley apparatus should easily be able to detect this difference.

• In fact, the result was the exact opposite: light always moves at the same speed regardless of the velocity of the source or the observer or the direction that the light is moving!
A Thought Experiment

• A person on a cart moving at half the speed of light will see light moving at c.
• A person watching on the ground will see that same light moving at the same speed, whether the light came from a stationary or moving source
I also see both beams moving at c! 0.8c

I also see both beams moving at c! 0.5c

I see both these light beams moving at c
So how is this possible??

• In the 18 years after the Michelson-Morley experiment, the smartest people in the world attempted to explain it away.

• In particular C.F. FitzGerald and H.A. Lorentz constructed a mathematical formulation (called the Lorentz transformation) which seemed to explain things but no one could figure out which it all meant.

• In 1905, Albert Einstein proposed the theory of Special Relativity which showed that the only way to explain the experimental result is to suppose that space and time as seen by one observer are distorted when observed by another observer (in such a way as to keep c invariant).
Welcome to The Strange World of Albert Einstein

• Some of the consequences of Special relativity are:
  – Events which are simultaneous to a stationary observer are not simultaneous to a moving observer.
  – Nothing can move faster than c, the speed of light in vacuum.
  – A stationary observer will see a moving clock running slow.
  – A moving object will be contracted along its direction of motion.
  – Mass can be shown to be a frozen form of energy according to the relation $E=mc^2$. 
Events

• In physics jargon, the word event has about the same meaning as it’s everyday usage.

• An event occurs at a specific location in space at a specific moment in time:
Reference Frames

• A reference frame is a means of describing the location of an event in space and time.
• To construct a reference frame, lay out a bunch of rulers and synchronized clocks.
• You can then describe an event by where it occurs according to the rulers and when it occurs according to the clocks.
Lorentz Transformation

• Space and time are not absolute as in Newtonian physics and everyday experience.

• The mathematical relation between the description of two different observers is called the Lorentz transformation.

• Some phenomena which follow from the Lorentz transformation are:
  – Relativity of Simultaneous events
  – Time Dilation
  – Length Contraction
More on Reference Frames

• What is the relation between the description of an event in a moving reference frame and a stationary one?
• To answer this question, we need to use the two principles of relativity.
The First Principle of Relativity

• An inertial frame is one which moves through space at a constant velocity

• The first principle of relativity is:
  – The laws of physics are identical in all inertial frames of reference.

• For example, if you are in a closed box moving through space at a constant velocity, there is no experiment you can do to determine how fast you are going

• In fact the idea of an observer being in motion with respect to space has no meaning.
The Second Principle of Relativity

• The second principle of relativity is a departure from Classical Physics:
  – The speed of light in vacuum has the same value, \( c \), in all inertial frames regardless of the source of the light and the direction it moves.

• This is what the MM experiment shows.
• The speed of light is therefore very special
• This principle is not obvious in everyday experience since things around us move much slower than \( c \).
• In fact, the effects of relativity only become apparent at high velocities
I also see both beams moving at c!

0.8c

I also see both beams moving at c!

0.5c

I see both these light beams moving at c.
What Happens to Simultaneous Events?

• Are events which are simultaneous to one observer also simultaneous to another observer?
• We can use the principles of relativity to answer this question.
• Imagine a train moving at half the speed of light…
A flash of light from the middle of the train sets off the fire crackers at either end simultaneously according to an observer on the train.
The observer watching the train go by sees the flash of light expand about the origin of the pulse and so she sees the rear firecracker explode first.
Simultaneous Events

• Thus two events which are simultaneous to the observer on the train are not simultaneous to an observer on the ground
• The rearwards event happens first according to the stationary observer
• The stationary observer will therefore see a clock at the rear of the train ahead of the clock at the front of the train
According to the person on the train, the two clocks are synchronized.

According to the stationary observer, the rear clock runs ahead of the front clock.
Of course it works the other way also. The observer on the train sees the stationary observer moving left at 0.5c. The person on the train sees the right hand stationary clock clock running ahead!
Time Dilation

• Consider the relation between time as measured by moving and stationary observers.
• To measure time, use a light clock where each “tick” is the time it takes for a pulse of light to move a given distance.
Time Dilation (cont.)

- Now let us imagine a train passing a stationary observer where each observer has an identical light clock.
- The observer on the train observes his light clock working normally each microsecond the clock advances one unit as the light goes back and forth:
Time Dilation (cont.)

• Now what does the stationary observer see?

• Compared to a stationary observer, the light beam travels quite far. Thus each tick of the moving clock corresponds to many ticks of the stationary clock.
Of course it works the other way also. The observer on the train sees the stationary observer moving left at 0.5c. The person on the train sees the right hand stationary clock running ahead!
So How Much Does The Moving Clock Run Slow?

• Let $t_0$ be the time it takes for one tick according to someone on the train and $t$ be the time according to someone on the ground.

• From what we just discussed $t > t_0$ but by how much?……

• The factor ($\gamma$) quantifies the amount of time dilation at a given velocity.
What the stationary observer sees happen as time $t$ passes.

What the observer on the train sees in as time $t_0$ passes for him.

Putting these lengths together,

$$(ct)^2 = (vt)^2 + (ct_0)^2$$

$$t = \frac{t_0}{\sqrt{1-(v/c)^2}}$$
The Factor called Gamma

• Thus, the time recorded on the moving clock, is related to the time that the stationary clock records according:

\[ t = \frac{t_0}{\sqrt{1 - (v/c)^2}} \]

• For simplicity we write the relation as:

\[ t = \gamma \ t_0 \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \]
## Some Time Dilation Factors

<table>
<thead>
<tr>
<th>Velocity</th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space Shuttle 5000m/s</td>
<td>1.000000000036</td>
</tr>
<tr>
<td>Earth in Orbit 30000m/s</td>
<td>1.0000000047</td>
</tr>
<tr>
<td>0.01c</td>
<td>1.00005</td>
</tr>
<tr>
<td>0.1c</td>
<td>1.005</td>
</tr>
<tr>
<td>0.5c</td>
<td>1.15</td>
</tr>
<tr>
<td>0.8c</td>
<td>1.67</td>
</tr>
<tr>
<td>0.9c</td>
<td>2.29</td>
</tr>
<tr>
<td>0.95c</td>
<td>3.20</td>
</tr>
<tr>
<td>0.99c</td>
<td>7.09</td>
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<td>0.999c</td>
<td>22.37</td>
</tr>
<tr>
<td>0.9999c</td>
<td>70.7</td>
</tr>
</tbody>
</table>
Time Dilation (cont.)

• For example, suppose that a rocket ship is moving through space at a speed of 0.8c.
• According to an observer on earth 1.67 years pass for each year that passes for the rocket man, because for this velocity gamma=1.67
• But wait a second! According to the person on the rocket ship, the earth-man is moving at 0.8c. The rocket man will therefore observe the earth clock as running slow!
• Each sees the other’s clock as running slow. HOW CAN THIS BE!!!!!
FitzGerald Length Contraction

• Just as relativity tells us that different observers will experience time differently, the same is also true of length.
• In fact, a stationary observer will observe a moving object shortened by a factor of $\gamma$ which is the same as the time dilation factor.
• Thus, if $L$ the length of an object as seen by a stationary observer and $L_0$ is the length in the moving frame then:

$$L = L_0 / \gamma$$
Why Length Contraction?

- Suppose that a rocket moves from the Sun to the Earth at $v = 0.95c$ ($= \gamma \cdot 2$).
- According to an observer from Earth, the trip takes 500s.

- By time dilation, only $500s / 3.2 = 156s$ pass on the ship. The crew observes the Earth coming at them at 0.95c.
- This means that the sun-earth distance according to the crew must be reduced by 3.2!

- Ship covers 150,000,000 km in 500 s
- Earth covers 47,000,000 km in 156 s
The Twin Paradox

• To bring this issue into focus, consider the following story:
  – Jane and Sally are identical twins. When they are both age 35, Sally travels in a rocket to a star 20 light years away at \( v = 0.99c \) and the returns to Earth. The trip takes 40 years according to Jane and when Sally gets back, Jane has aged 40 years and is now 75 years old. Since \( \gamma = 7.09 \), Sally has aged only 5 years 8 months and is therefore only 40 years and 8 months old. Yet according to the above, when Sally was moving, she would see Jane’s clock as running slow. How is this possible???
40 years later
On the outwards trip, the moving observer sees a row of lab clocks rush past him at velocity $v$. To him each clock runs slow but each clock he encounters is set ahead of the previous one. The same is true on his trip back so when he arrives back at Earth, the earth clock has advanced 6 years while his clock only advances 2 years.
Paradox

• Another way of thinking about the situation is as follows:
  – If two observers move past each other, each sees the other’s clock as moving slow.
  – The apparent problem is resolved by the change in time with position.
  – In the case of the twin paradox, there is not a symmetric relation between the two twins.
  – The earthbound twin was in an inertial frame the whole time.
  – The traveling twin underwent an acceleration when she turned around and came back. This breaks the symmetry between the two.
The Concept of Space-time

• Recall that an event takes place at a specific point in space at a specific time.
• We can therefore think of an event as a point in space-time.
• It is conventional to display time as a vertical axis and space as the horizontal axis.
Space-Time Diagrams

• Every event can be represented as a point in space-time
• An object is represented by a line through space-time known as it’s “world line”
• If we label the axes in natural units, light moves on lines at a 45° angle
An Object standing still

An Object Moving

A piece of light

The light cone

Time (in seconds)

Position (in It-seconds)
A Train Standing Still

Time

Position
A Train Moving
The speed of light

The second principle of relativity implies that you can never catch up to a piece of light, therefore you cannot accelerate through the “light barrier”

If there did exist a magic bullet that could travel faster than light, it would imply that you could travel or at least send information back in time

Thus an event can only effect what lies in its future light cone and can only be effected by events in its past light cone

The Moving finger writes; and, having writ,
Moves on: nor all thy piety nor wit
Shall lure it back to cancel half a line,
Nor all thy tears wash out a word of it.

-Omar Khayyam
A Train Standing Still

Magic Bullet
A trip to the Stars

• Consider a space ship which
  – accelerates at 1g for the first half of the trip
  – decelerates at 1g for the second half of the trip
• At this acceleration one can achieve speed near the speed of light in about a year
  – At 1 year of acceleration v=0.761 c
• In fact, within the life time of the crew, one could reach the edges of the universe!!!
**Acceleration/Deceleration = 1 g**

<table>
<thead>
<tr>
<th>Distance (ly)</th>
<th>Ship time (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3.5</td>
</tr>
<tr>
<td>100</td>
<td>9.2</td>
</tr>
<tr>
<td>30,000</td>
<td>20.61</td>
</tr>
<tr>
<td>2,000,000</td>
<td>29.01</td>
</tr>
</tbody>
</table>

**Graph**:
- **Time (in years)**
- **Position (in lt-years)**
- **Distant Star**
- **Deceleration**
- **Turnaround**
- **Acceleration**
Energy

- Since the speed of light is the ultimate speed limit
- If you accelerate an object towards c, it’s velocity gets closer to c but never reaches it
- The amount of energy required to do this is thus greater than \( \frac{1}{2}mv^2 \)
- In fact, \( K = (\gamma - 1)mc^2 \)
- Einstein realized that to have a meaningful definition of Energy which is connected to the geometry of space-time it is necessary to assign an energy \( E_0 = mc^2 \) to an object at rest.
- Thus, the total energy of an object including its rest energy and kinetic energy is \( E_{rel} = E_0 + K = mc^2 \gamma \)
General Relativity

• Magnetism and time dilation
• Gravity and Curved space-time
  – Curved in What
• Black holes
• The Big Bang
Energy

- Since the speed of light is the ultimate speed limit
- If you accelerate an object towards c, it’s velocity gets closer to c but never reaches it
- The amount of energy required to do this is thus greater than ½mv²
- In fact \( K = (\gamma - 1)mc^2 \)
- Einstein realized that to have a meaningful definition of Energy which is connected to the geometry of space-time it is necessary to assign an energy \( E_0 = mc^2 \) to an object at rest.
- Thus, the total energy of an object including its rest energy and kinetic energy is \( E_{\text{rel}} = E_0 + K = mc^2 \gamma \)
Relativity and Magnetism

- Imagine that someone holds two + charges near each other on a train moving near the speed of light.
- The person on the train sees the two charges moving apart at an acceleration $a$.
- His clock, however, runs slow according to an observer on the ground so the stationary observer sees them accelerate at a lesser acceleration.
- The stationary observer thus thinks there is an attractive force reducing the coulomb repulsion.
Relativity and Magnetism (cont.)

• Relativity thus requires that moving charges or currents will experience a force according to a stationary observer.
• The easiest way to think of this is to introduce the concept of a magnetic force.
The Equivalence Principle

• The cornerstone of General relativity is the Equivalence principle:

Gravitation and acceleration are equivalent: No experiment in a small box can tell the difference between acceleration and a uniform gravitational field.

Conversely, free fall is indistinguishable from the absence of gravity.
General Relativity

• Thus, to extend the concepts of Special Relativity to General Relativity Einstein modified the first principle of relativity to include the Equivalence principle thus
  The laws of physics are identical in all inertial frames of reference.

• Becomes
  The laws of physics are identical in all sufficiently small inertial frames of reference in free fall.
Why Curvature?

• On a curved surface, small regions look flat.
• For example people used to think that the earth was flat since you can’t see the curvature if you look on a small scale
• Likewise in a small box, you can’t tell whether you are in free fall or in empty space.
• On a curved surface, two lines, initially parallel may cross. Likewise a brick, initially moving through time parallel to the earth eventually strikes the earth.
Lensing of distant galaxies by a nearby cluster of galaxies
Black Holes

- As an object (e.g. star) becomes more compact, the velocity required to escape the surface becomes greater and greater.
- When this velocity becomes $c$, it is no longer possible to escape the gravitation pull and the object becomes a black hole.
- For instance, the earth compressed to 1.5 cm or the sun compressed to 1.4 km.
- The curvature of space-time is so drastic near a black hole that strange things start to happen.
Gravity and Time

• A clock close to a massive object will seem to run slow compared to someone far from the object (normally this effect is too small to easily measure as with special relativistic effects)

• So what happen if you fall into a black hole? Suppose that Bill C. falls into a Black hole and Al G. remains far form the BH (and thus becomes president)
What Al and Bill see

• What Al G. sees
  – Bill approaches the EVENT HORIZON, his clock runs slow, he becomes red.
  – He never hits the event horizon, Al G. could in principle rescue him but this becomes harder in practice as time goes on.
  – Also, as Bill approaches the event horizon, he appears to be flattened, similar to Fitzgerald contraction.

• What Bill C. sees
  – He sees Al’s clock moving faster and faster. It hits infinity when he crosses the event horizon.
  – It then reverses as he passes the EH. Bill is now within the Black Hole and cannot escape.
  – Time and space are swapped for him, as he moves forwards in time, he moves towards the center of the black hole. He cannot avoid it.
  – Eventually he hits the singularity at the center of the BH. He ceases to exist.
Dust orbiting a black hole

• This black hole is a billion times the mass of the sun and the size of the solar system.
• It is 100,000,000 ly away.
• You can’t see the black hole directly but a dust cloud 800 ly across orbits it.
Core of Galaxy NGC4261
PRC95-47 · ST ScI OPO · December 4, 1995
H. Ford and L. Ferrarese (JHU), NASA
Black Holes

• Kinds of Black Holes we know are out there
  – Stellar black holes, the remains of dead stars which are too massive to form neutron stars or white dwarfs. Masses are a few X the mass of the sun
  – Super Massive Black Holes at the core of galaxies which are a million to a billion solar masses. Most galaxies have one including our own.
Curved in What?

- If gravity results from the curvature of space-time, it seems natural to ask what space-time is curved in.
- It is mathematically possible that curvature is just an intrinsic property of space, however…
- Some physicists speculate that there may be up to 7 more “short” dimensions which have yet to be observed.
Again, the special theory of relativity changes our conceptions of space and time.

As seen by the outfielder, the ball is approaching her at \((30 \text{ m/s}) + (10 \text{ m/s}) = 40 \text{ m/s}\).

- This theory, published by Einstein in 1905, is based on the notion that there is no such thing as absolute space or time.
- Space and time are not wholly independent of each other, but are aspects of a single entity called spacetime.
The speed of light is the same to all observers, no matter how fast they are moving.

Incorrect Newtonian description:
As seen by the astronaut in spaceship, the light is approaching her at $(3 \times 10^8 \text{ m/s}) + (1 \times 10^8 \text{ m/s}) = 4 \times 10^8 \text{ m/s}$.

Correct Einsteinian description:
As seen by the astronaut in spaceship, the light is approaching her at $3 \times 10^8 \text{ m/s}$.
An observer will note a slowing of clocks and a shortening of rulers that are moving with respect to the observer. This effect becomes significant only if the clock or ruler is moving at a substantial fraction of the speed of light.
The general theory of relativity is our most accurate description of gravitation

- Published by Einstein in 1915, this is a theory of gravity
- A massive object causes space to curve and time to slow down
- These effects manifest themselves as a gravitational force
- These distortions of space and time are most noticeable in the vicinity of large masses or compact objects
(a) The apple hits the floor of the compartment because the Earth’s gravity accelerates the apple downward.

(b) The apple hits the floor of the compartment because the compartment accelerates upward.
The theory of relativity predicts a number of phenomena, including the bending of light by gravity and the gravitational redshift, whose existence has been confirmed by observation and experiment.
• The general theory of relativity also predicts the existence of gravitational waves, which are ripples in the overall geometry of space and time produced by moving masses

• Gravitational waves have been detected indirectly, and specialized antennas are under construction to make direct measurement of the gravitational waves from cosmic cataclysms
The general theory of relativity predicts black holes

1. A supergiant star has relatively weak gravity, so emitted photons travel in essentially straight lines.
2. As the star collapses into a neutron star, the surface gravity becomes stronger and photons follow curved paths.
3. Continued collapse intensifies the surface gravity, and so photons follow paths more sharply curved.
4. When the star shrinks past a critical size, it becomes a black hole: Photons follow paths that curve back into the black hole so no light escapes.
• If a stellar corpse has a mass greater than about 2 to 3 \( M_\odot \), gravitational compression will overwhelm any and all forms of internal pressure.

• The stellar corpse will collapse to such a high density that its escape speed exceeds the speed of light.
Certain binary star systems probably contain black holes

- Black holes have been detected using indirect methods
- Some binary star systems contain a black hole
- In such a system, gases captured from the companion star by the black hole emit detectable X rays
1. Gases from the supergiant are captured into an accretion disk around the black hole.

2. As gases spiral toward the black hole, they are heated by friction: Just outside the black hole, they are hot enough to emit X rays.

A schematic diagram of Cygnus X-1
An artist’s impression of Cygnus X-1
This black hole is surrounded by a disk of hot gas.

Torus (doughnut) of cooler gas and dust (shown cut away)

Fast-moving jets of subatomic particles are formed and ejected by electric and magnetic fields.

X rays from the hot disk excite iron atoms in the torus, making them glow.
Supermassive black holes exist at the centers of most galaxies. These are detected by observing the motions of material around the black hole.
A nonrotating black hole has only a “center” and a “surface”

- The entire mass of a black hole is concentrated in an infinitely dense singularity.
- The singularity is surrounded by a surface called the event horizon, where the escape speed equals the speed of light.
- Nothing—not even light—can escape from inside the event horizon.
Just three numbers completely describe the structure of a black hole

- A black hole has only three physical properties: mass, electric charge, and angular momentum.
- A rotating black hole (one with angular momentum) has an ergoregion around the outside of the event horizon.
- In the ergoregion, space and time themselves are dragged along with the rotation of the black hole.
(a) Looking directly toward the black hole from a distance of 1000 Schwarzschild radii: Note positions of stars 1, 2, and 3.

(b) Looking directly toward the black hole from a distance of 10 Schwarzschild radii: Light bending causes multiple images.
Falling into a black hole is an infinite voyage
Could a black hole somehow be connected to another part of spacetime, or even some other universe?

General relativity predicts that such connections, called wormholes, can exist for rotating black holes.
Black holes evaporate

1. Pairs of virtual particles spontaneously appear and annihilate everywhere in the universe.

2. If a pair appears just outside a black hole’s event horizon, tidal forces can pull the pair apart, preventing them from annihilating each other.

3. If one member of the pair crosses the event horizon, the other can escape into space, carrying energy away from the black hole.
Jargon

- black hole
- black hole evaporation
- equivalence principle
- ergoregion
- event horizon
- general theory of relativity
- gravitational radiation
- gravitational waves
- gravitational redshift
- Heisenberg uncertainty principle
- law of cosmic censorship
- length contraction
- Lorentz transformations
- mid-mass black hole
- no-hair theorem
- primordial black hole
- proper length (proper distance)
- proper time
- Schwarzschild radius ($R_{\text{Sch}}$)
- singularity
- spacetime
- special theory of relativity
- stellar-mass black hole
- supermassive black hole
- time dilation
- virtual pairs
- wormhole