This Class (Lecture 6):
Laws of Motion

Next Class:
Gravitation

**HW #2 Due Friday!**

**New TA Office Hours**

**Stardial Observing is available**

**Music:** *World by a String* – Neil Young

---

**Isaac Newton (1642-1727)**

**Newton's 1st Law of Motion**

- **Law of Inertia**

  “An object at rest will remain at rest and an object in motion will remain in motion in a straight line at constant speed, unless acted upon by an unbalanced force.”

- **Motion is described by velocity, which measures speed and direction**
Restated

- Free body (no net force), then velocity is constant.
- Remember, velocity is a vector!
- Encodes Galileo's "free body" behavior
- Establishes existence of inertial frames
- Or we can say, that the momentum of an object remains constant unless it experiences an unbalanced force.

What is a Force?

- No, not THE Force…
- Force in the simplest sense is a push or pull. It may be from gravity, electrical, magnetic, or muscle efforts
- Measured in Newtons

Why was it so hard to see this?

- Usually we have Friction!
- Friction is a possible net outside force that Newton was talking about!
- Remember the feather/hammer experiment? Air Friction dominates the feather causing this to fail in the classroom.

Newton's 2nd Law of Motion

- Law of Acceleration
  - The net force acting on an object is proportional to the object's mass and its resulting acceleration.
  - \( a = \frac{F}{m} \) or \( F = ma \)
- Acceleration is a change in velocity (in speed and/or direction, think of the 1st law)
  - Measured in meters per second per second
  - To accelerate something you have to apply a force
- Mass is amount of matter in an object
  - Measured in grams or kilograms, not pounds!
**Elephant at rest stays at rest**

Takes a big force, or the Elephant stays at rest. Or an anvil in space— even if it is “weightless”.

http://sol.sci.uop.edu/~jfalward/physics17/chapter2/chapter2.html

---

**Newton's 2nd Law of Motion**

\[ \vec{F}_{\text{net}} = \sum_{i=1}^{n} \vec{F}_i = m\vec{a} \]

\[ \vec{F}_{\text{net}} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt} \]

---

**Newton's 3rd Law of Motion**

- Law of Action-Reaction
  - “Every action has an equal and opposite reaction”
  - Action: Guy jumps forward out of the boat
  - Reaction: Boat moves away from the pier

**Newton's 3rd Law of Motion**

- Law of Action-Reaction
  - “Every action has an equal and opposite reaction”
  - Action: Player makes a shot.
  - Reaction: He moves backwards slightly.
Newton's 3rd Law of Motion

\[ \vec{F}_{12} = -\vec{F}_{21} \]

Equal Forces— and no acceleration

http://www.ac.wwu.edu/~vawter/PhysicsNet/Topics/Dynamics/ThirdLaw.html

http://sol.sci.uop.edu/~jfalward/physics17/chapter2/chapter2.html

Recap: Newton's Laws of Motion

- The Law of Inertia
  - If a body is left to itself, it will move in a straight line forever

- The Law of Acceleration
  - Acceleration = Force ÷ Mass

- The Law of Action-Reaction
  - For every action, there is an equal and opposite reaction
Newton and the Planets

This can/should be applied to the heavens as well as the Earth. Right?

Planetary Motion… By Newton

- A planet going around the Sun (or a moon going around a planet) is always accelerating
  - The direction of motion is changing
- There must be a force acting on the planet!
  - Imagine it as a string
- If we “cut the string”, what happens?  
  - According to Newton’s 1st Law, the ball moves in a straight line

Nature of Gravity

- Newton's Law of Acceleration then tells us that the Sun MUST be applying a force

Newton's Law of Gravity

Two bodies attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
Newton's Law of Gravity

- Any two masses have a gravitational force between them:
  \[ F = G \frac{m_1 m_2}{r^2} \]
  - \( m_1 \) and \( m_2 \) are the masses
  - \( r \) is the distance between the 2 masses
  - \( G \) is the "gravitational constant"

Inverse Square Law

- Gravity is what we call an "Inverse Square Law"
- Strong function of separating distance!
- Doubling the distance quarters the force!

Gravity

\[ G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^2 \]

Newton's Great Insight

- The same force makes things fall down on Earth and keeps the planets in their orbits

Orbiting bodies are falling bodies!

Or
http://spaceplace.jpl.nasa.gov/orbits1.htm
Newton’s Universal Law of Gravity

The Earth pulls you and you pull it. But the Earth wins, inertial-wise.

\[ F = m \cdot a \]

Why was this important?

- Remember, the ancients believed that there were two sets of rules
  - One for Earth
  - One for the Heavens
- Newton showed that the same laws of nature applied everywhere!
- Earth is not a “special place”
- We are a part of the Universe

What is Weight?

- What we feel as weight is actually the force we feel from Newton’s Law of Gravity.

\[
\text{Weight} = \frac{G M_{\text{Earth}} M_{\text{you}}}{R_{\text{Earth}}^2}
\]

- It is confusing since social convention has made weight and mass the same at the earth’s surface, but what happens to your weight elsewhere?
Gravity on the Moon

• Is there gravity on the moon?

• Yes!
  • But the Moon is around 3.7 times smaller and has 81 times less mass than the Earth
  • The result is that your weight is $1/6^{th}$ of your Earth weight

Gravity Bringing You Down?

From $F = ma$:
where $m$ is a test mass

Falling Objects

• $g$ is the same for every object!
• The ancients thought heavy objects fall faster
  – A hammer falls faster than a feather, right?
  – Neglected air resistance
• In the absence of air resistance, the hammer and the feather fall with the same acceleration

“Zero-G”

• Why are astronauts “weightless” when in orbit? Are they out of the Earth's gravity?
  – No! Gravity is what keeps them in orbit
  – Astronauts feel weightless because they are falling at the same speed as the spacecraft
  – There is no force pressing them against the floor
Free Falling Objects

“weightless”: not beyond influence of gravity

- Astronaut is just another orbiting body
- Earth’s pull is what keeps astronaut in orbit
- Astronaut feels “weightless” because she and spacecraft are experiencing gravity together

Gravity Bringing You Down?

Shuttle orbit is 300 km and Earth’s radius is 6378 km

\[ a = G \frac{M_\oplus}{(R_\oplus + R_{\text{orbit}})^2} = 8.9 \frac{\text{m}}{\text{s}^2} \]

- Force of gravity on astronaut is nearly the same as on Earth
- Not really weightless at all.
- Astronaut and the space shuttle are both in free-fall with identical accelerations.
- The amount one “falls” towards center of earth changes position/velocity to match the change in position/velocity required for circular motion.

Newton's Orbits

- Kepler had produced his three laws by trying to fit mathematical formulae to his data
- Newton found he could derive Kepler’s Laws from his Law of Gravity and Laws of Motion!

Circular Orbit

- This is accelerated motion!
- Velocity direction is tangent to the path
- Direction of velocity changes as object rotates \( \rightarrow \) acceleration
- Change in velocity is perpendicular to velocity
- Acceleration direction points towards circle center
Circular Motion Review

- Angular Velocity:

- Position: \[ x = r \cos(\omega t) \]
  \[ y = r \sin(\omega t) \]

- Velocity \[
  \begin{align*}
  \dot{x} &= \omega r \cos(\omega t) \\
  \dot{y} &= -\omega r \sin(\omega t)
  \end{align*}
\]

- Acceleration \[
  \begin{align*}
  \ddot{x} &= -\omega^2 r \cos(\omega t) \\
  \ddot{y} &= -\omega^2 r \sin(\omega t)
  \end{align*}
\]

Angular Momentum

What is \( \vec{r} \times \vec{F} \)? \[ \vec{L} = \vec{r} \times m \vec{v} \]

What about gravity guaranteed that \( \frac{d\vec{L}}{dt} = 0 \)?

Conserved!!!!

Kepler's 1st Law by Newton

- Newton also found more options that satisfied his universal law of gravity

- For any attractive inverse square law force, can show that the orbits are conic sections with the other body at one focus.

![Diagram of conic sections: Circle, Ellipse, Parabola, Hyperbola.](image)