Summary of Radiometric Terms

**Radiant flux (W):** the amount of radiant energy emitted, transmitted, or received per unit time.

**Radiant flux density (W/m²):** radiant flux per unit area

**Irradiance (W/m²):** radiant flux density incident on a surface

**Radiant spectral flux density (W m⁻² μm⁻¹):** radiant flux density per unit of wavelength interval.

**Radiant intensity (W/sr):** flux emanating from a surface per unit solid angle.

**Radiance (W m⁻² sr⁻¹):** radiant flux density emanating from a surface per unit solid angle

**Spectral radiance (W m⁻² sr⁻¹ μm⁻¹):** radiance per unit wavelength interval.

**Radiant emittance (W/m²):** radiant flux density emitted by a surface.
Summary of radiometric terms

Radiant energy (J)

Add time

Radiant flux (J/S = W)

**Hemispherical**

Add area

Radiant flux density (W/m²)

Irradiance (incident)

Radiant emittance (emitted)

Add wavelength

Radiant spectral flux density (W m⁻² μm⁻¹)

**Directional**

Add area

Radiant intensity (W/sr)

Add direction

Radiance (W m⁻² sr⁻¹)

Add wavelength

Spectral radiance (W m⁻² sr⁻¹ μm⁻¹)
### Reflectance terms

**Bi – directional reflectance:** the ratio of the reflected radiance from a single view direction to the irradiance from some incident view direction that is confined to a very narrow range of incident angles.

**Directional – hemispherical reflectance:** the ratio of the reflected radiance integrated over the entire view hemisphere to the irradiance from a single view direction confined to a narrow range of incident angles.

**Hemispherical – directional reflectance:** ratio of the reflected radiation from a single view direction to the incident irradiance averaged over the entire incident hemisphere.

**Bi – hemispherical reflectance:** the ratio of the reflected radiance integrated over the entire view hemisphere to the incident irradiance averaged over the entire incident hemisphere of incoming radiation.

**Bi – directional reflectance factor:** is the ratio of the reflected radiance from a single view direction to the reflected radiance from an ideal, perfectly diffuse surface experiencing the same irradiance (reference panel).
The Cosine Law

\[ M = M_0 \cos \theta \]

Where \( M_0 \) is the flux density normal to the beam, \( M \) is the flux density at the surface, and \( \theta \) is the angle between the radiant beam and a normal to the surface which is referred to as the zenith angle.
Irradiance

For isotropic radiation with radiance of $L$ (radiance is constant across all incident directions in the hemisphere):

$$E_\lambda = \int_0^{2\pi} \int_0^{\pi/2} L_\lambda \sin \theta \cos \theta d\theta d\psi = \pi L_\lambda$$

Where $\theta$ is the zenith angle, and $\psi$ is the azimuth angle.

Thus the irradiance of a surface under isotropic radiation is always $\pi$ times the radiance.
Radiation Components

Surface reflected

Unscattered  Down-scattered  Path-scattered
Radiation Components

\[ L^s_\lambda = L^{su}_\lambda + L^{sd}_\lambda + L^{sp}_\lambda \]
Surface Reflected Unscattered Component

\( E_\lambda^0 \) is the top of atmosphere irradiance. It can be calculated using Planck's blackbody equation and a few geometrical terms. It varies by only a couple of percent depending on the distance between the sun and earth.

\( E_\lambda \) is the irradiance at the surface of the earth and is dependent on the solar path atmospheric transmittance(\( \tau_s \)) as well as the zenith angle(\( \theta \)) by way of the cosine rule.

\[
E_\lambda(x, y) = \tau_s(\lambda) E_\lambda^0 \cos \theta(x, y)
\]
Surface Reflected Unscattered Component

The solar path atmospheric transmittance ($\tau_s$) is the variable difficult to determine. It is a function of the distance the solar beam travels through the atmosphere (which is a function of the solar zenith angle ($\theta$)) as well as atmospheric parameters which influence scattering, absorption, and transmittance. There are complex radiative transfer programs available to determine it such as MODTRAN. They require estimates of atmospheric parameters such as water vapor content, aerosol content, etc.. to be used.
Surface Reflected Unscattered Component

After calculating the irradiance at the earths surface, the next energy transfer occurs upon reflectance with a surface material. The irradiance downward onto a lambertian surface is converted to the radiance leaving the surface with the aid of a geometric factor $\pi$:

$$L_\lambda(x, y) = \rho(x, y, \lambda) \frac{E_\lambda}{\pi}$$
Surface Reflected Unscattered Component

Now we must account for the radiance leaving the surface and traveling through the atmosphere once again towards the sensor:

\[ L_{\lambda}^{su} = \tau_v(\lambda)L_{\lambda} \]

Thus the total surface reflected unscattered component is:

\[ L_{\lambda}^{su} = \rho(x, y, \lambda) \frac{\tau_v(\lambda)\tau_s(\lambda)E_{\lambda}^{0}}{\pi} \cos\left[ \theta(x, y) \right] \]
Radiation Components

Surface reflected

Unscattered  Down-scattered  Path-scattered
Surface Reflected Atmospheric Down-scattered Component

Why are shadows not completely dark?

Because of diffuse radiation scattered downwards by the atmosphere.

\[
L_{sd}^\lambda = F(x, y) \rho(x, y, \lambda) \frac{\tau_v(\lambda)E^d_\lambda}{\pi}
\]

Where \( F \) is the fraction of the sky hemisphere which is visible from the pixel of interest and \( E^d_\lambda \) is the diffuse sky irradiance.
Radiation Components

$E_\lambda^0$, $E_\lambda$, $L_{su}^\lambda$, $L_{sd}^\lambda$, $L_{sp}^\lambda$

Surface reflected

Unscattered, Down-scattered, Path-scattered
Path Scattered Component

The path scattered component is a function of the amount of Rayleigh, Mie, and non-selective scattering in the atmosphere. It is highly dependent on wavelength. It is assumed to be constant over a scene and can be determined using radiative transfer models.
Total At-Sensor Radiance

\[ L^s_\lambda = L^{su}_\lambda + L^{sd}_\lambda + L^{sp}_\lambda \]

\[ L^s_\lambda (x, y) = \rho(x, y, \lambda) \frac{\tau_v(\lambda)}{\pi} \left\{ \tau_s(\lambda)E^0_\lambda \cos[\theta(x, y)] + F(x, y)E^d_\lambda \right\} + L^{sp}_\lambda \]