QM, Final Exam, 2009: Take Home

(1) (30 points) Consider three non-interacting electrons in a two-dimensional harmonic potential, \( V(x, y) = x^2 + 2y^2 \). Calculate the ground state energy and wave function of the system. How does the answer change if (a) the particles are \( \pi^- \)? (b) the three particle system consists of an electron, proton and a neutron.

(2) (20 points) A particle in a spherically symmetric potential is described by a wave function, \( \psi(x, y, z) = A(r^2 + bz^2) \) where \( A \) is a normalization constant. Calculate the possible angular momentum quantum numbers of the system and the probability of being found in those states. How does your answer change if \( \psi(x, y, z) = B(r + y) \).

(3) (20 points) Find the \( <X> \), \( <P> \), \( \Delta X \) and \( \Delta P \) for the following wave function with normalization constant \( A \). \( \psi(p) = Ae^{ix}e^{-(p-1)^2} \)

(4) (10 points) Given \( f(x) = (x^2 - a^2) \), calculate \( \delta(f(0)) \).

(5) (10 points) A particle is described by a wave function \( \psi(x) = Alog(x^4)\sin(x)e^{-x^2}e^{5ix} \) where \( A \) is a normalization constant. Calculate \( <p> \).

(6) (10 points) Calculate the expectation value of \( <n|/(x/x_0 + p/p_0)^2|n> \) for a particle of mass unity, moving in potential
(7) (10 points) A particle is in the ground state of a cubic box of unit dimension. Suddenly, all the lengths change to half their values, leaving the wave function unchanged. Calculate the probability of finding the particle in the ground state of the new system.

(8) (20 points)
Exercise 11.4.2 (on page 300 of the book)

(9) (30 points)
Exercise 12.3.8 (on page 317-318 of the book)

(10) (20 points)
Exercise 7.4.6 (page 212) of the book.

\[ V(x) = 5(x - 1)^2, \] where \( x_0 \) and \( p_0 \) are constant.