9-6. Since particle 1 has $F = 0$, $r_0 = v_0 = 0$, then $r_1 = 0$. For particle 2

$$F_2 = F_0 \hat{x} \quad \text{then} \quad \ddot{r} = \frac{F_0}{m} \hat{x}$$

Integrating twice with $r_0 = v_0 = 0$ gives

$$r_2 = \frac{F_0}{2m} t^2 \hat{x}$$

$$r_{CM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \frac{F_0}{4m} t^2 \hat{x}$$

By symmetry $\bar{y} = 0$

$$m_o = 16 \ m_H$$

Let $m_H = m$, $m_o = 16 \ m$

Then $\bar{x} = \frac{1}{M} \sum m_i x_i$

$$\bar{x} = \frac{1}{18m} (2ma \cos 52^\circ) = \frac{a \cos 52^\circ}{9}$$

$$\bar{x} = 0.068 \ a$$

9-7. By symmetry $\bar{x} = 0$. Also, by symmetry, we may integrate over the $x > 0$ half of the triangle to get $\bar{y}$. $\sigma = \text{mass/area}$

$$\bar{y} = \frac{\int_{-x=0}^{a} \int_{y=0}^{\sqrt{2} \ x} \sigma \ y \ dy \ dx}{\int_{x=0}^{a} \int_{y=0}^{\sqrt{2} \ x} \sigma \ dy \ dx} = \frac{a}{3\sqrt{2}}$$

$$\bar{y} = \frac{a}{3\sqrt{2}}$$
Let the axes be as shown with the projectile in the y-z plane. At the top just before the explosion, the velocity is in the y direction and has magnitude \( v_{oy} = \frac{v_0}{\sqrt{2}} \).

\[
v_{oy} = \frac{v_0}{\sqrt{2}} = \sqrt{\frac{2E_0}{m_1 + m_2}} = \sqrt{\frac{E_0}{m_1 + m_2}}
\]

where \( m_1 \) and \( m_2 \) are the masses of the fragments. The initial momentum is

\[
p_i = (m_1 + m_2) [0, \sqrt{\frac{E_0}{m_1 + m_2}}, 0]
\]

The final momentum is

\[
p_f = p_1 + p_2
\]

\[
p_1 = m_1(0, 0, v_1)
\]

\[
p_2 = m_2(v_x, v_y, v_z)
\]

The conservation of momentum equations are

\[
p_x: \quad 0 = m_2v_x \quad \text{or} \quad v_x = 0
\]

\[
p_y: \quad \sqrt{\frac{E_0}{m_1 + m_2}} = m_2v_y \quad \text{or} \quad v_y = \frac{1}{m_2} \sqrt{\frac{E_0}{m_1 + m_2}}
\]

\[
p_z: \quad 0 = m_1v_1 + m_2v_z \quad \text{or} \quad v_1 = -\frac{m_2}{m_1} v_z
\]

The energy equation is

\[
\frac{1}{2}(m_1 + m_2) \cdot \frac{E_0}{m_1 + m_2} + E_0 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2(v_y^2 + v_z^2)
\]

or

\[
3E_0 = m_1v_1^2 + m_2(v_y^2 + v_z^2)
\]

Substituting for \( v_y \) and \( v_1 \) gives

\[
v_z = \sqrt{\frac{E_0 m_1(2m_2 - m_1)}{m_2^2(m_1 + m_2)}}
\]

\[
v_1 = -\frac{m_2}{m_1} v_z \text{ gives}
\]

\[
v_1 = -\sqrt{\frac{E_0 (2m_2 - m_1)}{m_1(m_1 + m_2)}}
\]
So \( m_1 \) travels straight down with speed \(|v_1|\);

\( m_2 \) travels in the \( y-z \) plane

\[
v_2 = (v_y^2 + v_z^2)^{1/2} = \sqrt{\frac{E_0 (4m_1 + m_2)}{m_2 (m_1 + m_2)}}
\]

\[
\theta = \tan^{-1} \frac{v_z}{v_y} = \tan^{-1} \frac{\sqrt{m_1 (2m_2 - m_1)}}{(m_1 + m_2)}
\]

The mass \( m_1 \) is the largest it can be when \( v_1 = 0 \), meaning \( 2m_2 = m_1 \) and the mass ratio is

\[
\frac{m_1}{m_2} = \frac{1}{2}
\]

9-10.

First, we find the time required to go from \( A \) to \( B \) by examining the motion. The equation for the \( y \)-component of velocity is

\[
v_y = v_0 \sin \theta - gt
\]

At \( B \), \( v_y = 0 \); thus \( t_B = \frac{v_0 \sin \theta}{g} \). The shell explodes giving \( m_1 \) and \( m_2 \) horizontal velocities \( v_1 \) and \( v_2 \) (in the original direction). We solve for \( v_1 \) and \( v_2 \) using conservation of momentum and energy.

\[
P_x: \quad (m_1 + m_2)v_0 \cos \theta = m_1v_1 + m_2v_2
\]

(2)

\[
E: \quad \frac{1}{2}(m_1 + m_2)v_0^2 \cos^2 \theta + E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2
\]

(3)

Solving for \( v_2 \) in (2) and substituting into (3) gives an equation quadratic in \( v_1 \). The solution is

\[
v_1 = v_0 \cos \theta \pm \sqrt{\frac{2m_2 E}{m_1 (m_1 + m_2)}}
\]

(4)

and therefore we also must have

\[
v_2 = v_0 \cos \theta \mp \sqrt{\frac{2m_1 E}{m_2 (m_1 + m_2)}}
\]

(5)

Now we need the positions where \( m_1 \) and \( m_2 \) land. The time to fall to the ocean is the same as the time it took to go from \( A \) to \( B \). Calling the location where the shell explodes \( x = 0 \) gives for the positions of \( m_1 \) and \( m_2 \) upon landing:

\[
x_1 = v_1 t_B; \quad x_2 = v_2 t_B
\]

Thus

\[
|x_1 - x_2| = \frac{v_0 \sin \theta}{g} \frac{1}{|v_1 - v_2|}
\]

(7)

Using (4) and (5) and simplifying gives

\[
|x_1 - x_2| = \frac{v_0 \sin \theta}{g} \sqrt{\frac{2E}{m_1 + m_2}} \left[ \sqrt{\frac{m_1}{m_2}} + \sqrt{\frac{m_2}{m_1}} \right]
\]

(8)