11 Ideal Gas Law and Heat Engine

Introduction
In this lab, you will explore the Ideal Gas Laws. In Part I, you will observe the proportional relationship between temperature and pressure, create a graph from data points, and use it to infer the proportionality constant. You will also determine the temperature for absolute zero. In Part II, you will use a heat engine apparatus to explore the relationship between temperature, pressure and volume. You will observe the conversion of heat into work and mechanical energy. This is a discovery lab, so there is no formal write up.

Materials
Part I: Data Studio, temperature sensor, pressure sensor, gas chamber, coffee pot, cold water/ice, paint stirrer
Part II: Data Studio, rotary motion sensor, pressure sensor, heat engine, cold water/ice bath, hot water bath

Reference
Giancoli, Physics 6th Edition: Chapter 13, sections: 1,2,4,6,7,9,10; Chapter 15, sections: 1,2,5

Theory
Part I: The ideal gas law relates the pressure, $P$, volume, $V$, and absolute temperature, $T$, (in Kelvin) of an ideal gas

$$PV = nRT \quad (1)$$

where $n$ is the number of moles of gas in the volume and $R$ is the Ideal Gas Constant (equal to 8.31 J/mol-K). We can rewrite Equation (1) as

$$P = \frac{nR}{V} T = CT$$, \quad (2)$$

where $C = nR/V$ is a constant in the case where volume is fixed. In summary, with fixed volume we expect:

$$P = CT$$, \quad (3)$$
We thus expect a linear graph that passes through the origin if the temperature units are Kelvin or absolute temperature. That is, when the pressure is zero, the absolute temperature will be zero.

The constant $C$ can be determined since we know that there will be 22.4 liters of volume per mole of gas at STP, i.e. $n = \frac{V}{22.4 \, \ell/\text{mole}}$. Assuming our volume is filled with gas at approximately STP, we get

$$C = \frac{n R}{V} = \frac{V R}{22.4 \, \ell/\text{mole}} = \frac{8.314 \, \text{Pa} \cdot \text{m}^3/\text{mole} \cdot \text{K}}{0.0224 \, \text{m}^3/\text{mole}} = 371 \, \text{Pa/K} \quad (4)$$

In the lab, you will use a fixed volume of gas, vary its temperature, and record the pressure. You will then plot $P$ versus $T$, where $T$ is in Celsius units, and extrapolate the straight line fit to find where it intersects the $x$-axis, i.e. where the pressure is zero, to determine the point at which the absolute temperature should be zero. This point, in Celsius, should equal absolute zero. Absolute zero is the temperature at which the pressure of the gas extrapolates to zero, meaning that the molecules have the lowest kinetic energy possible.

**Part II:** The heat engine cycle is a repeatable, closed thermodynamic cycle that is often represented by a pressure-volume diagram shown in Figure 12.1 below:

![Figure 12.1: Heat Engine Cycle](image)

The cycle has four transitions. In the horizontal segments, $d \rightarrow a$ and $b \rightarrow c$, the volume changes due to changes in temperature while pressure remains constant. During the other two steps, $a \rightarrow b$ and $c \rightarrow d$, the volume changes due to changes in the external pressure.
on the piston while the temperature is essentially constant. The heat engine used in this experiment uses a gas confined to a cylinder with a moveable piston plus a fixed chamber connected to the cylinder by flexible tubing. When the heat engine undergoes a transition at a constant pressure $P$ and the volume changes by the amount $\Delta V$ as in the horizontal segments above, the work done by the engine is $W = P \Delta V$. That is, work is done when the volume changes. If the volume increases, the work done by the gas in the heat engine is positive and when the volume decreases the work done by the engine is negative.

To see how the engine will work, review the picture above and the equipment at your table (shown in Figure 12.2 below). You will start by placing the chamber connected to the cylinder into a cold water bath. This will lower the temperature of the gas while the pressure will remain constant because the piston will adjust and allow the volume to contract (decrease). The system will then be at Point “a” in Figure 12.1.

Next, you will place a mass on the piston pushing the piston down, causing the volume to decrease and the pressure to increase. This will bring the system to Point “b”.

With the mass still on the cylinder, you will place the chamber in a hot water bath, causing the volume of gas to expand, raising the piston. This will cause the temperature to increased, while the pressure will be constant. This brings you to Point “c”.

Finally, you will remove the mass from the piston. This will decrease the pressure and allow the volume to expand, putting the system at Point “d”.

To return the system to the beginning of the cycle, you will return the chamber to the cold water bath. The temperature and volume will both decrease, returning us to Point “a”.

We will next examine the work done by the system. The change in volume is directly proportional to the change in position $\Delta x$ as measured by the motion sensor. We can measure the net work done over the cycle by measuring the area of the enclosed parallelogram. To find this area, we must find the area under each of the horizontal segments and subtract the area under the lower segment from the area under the upper. In essence, we add the negative work done when the volume decreases from $d \rightarrow a$ (transfer from hot to cold bath) to the positive work done when volume increases from $b \rightarrow c$ (transfer from cold to hot bath). For each transition, $P\Delta V = W$, and therefore, $W_{net} = P_1\Delta V_1 + P_2\Delta V_2$. Because the quantity $\Delta V_2$ in this case is negative, we find the difference between the two work values. The work done by raising the mass is also equal to the change in potential energy, $mg\Delta h$, where $\Delta h$ is the distance the piston has moved.
Procedure

Part I: Temperature and Pressure

1. Connect the pressure sensor to the chamber. The piston-and-cylinder unit is not used in this part and should not be connected to the chamber.

2. Set up the pressure (absolute pressure) and temperature sensors in Data Studio. Set the data acquisition frequency to 1 Hz for both sensors.

3. Fill the coffee pot with very cold water and some ice. Make sure to leave enough room for the chamber and the temperature sensor.

4. Put a paint stirrer in the coffee pot.

5. Set up the graph to record changes in temperature and pressure in the chamber. Graph the data with the pressure on the vertical axis and temperature on the horizontal axis. To do this, open the Graph Display from the list of "Displays". Now drag the Temperature Sensor from the "Data" list to the x-axis of the graph. Do the same thing for the Pressure Sensor and drag it to the y-axis. Click on “start” to start the data collection.

6. Turn on the coffee pot. You should be continuously collecting data while the water is heating and stirring the water at the same time. When 60°C is reached click on “stop” to stop the data collection and turn off the coffee pot.
7. Using the linear fit; highlight the best data for fitting. Using the equation provided, determine the $x$-intercept and estimate its uncertainty. The $x$-intercept is absolute zero in degrees Celsius, while the slope should equal $-\frac{nR}{V}$ as indicated by Equation (3).

8. Compare your experimentally determined $x$-intercept with the accepted value of absolute zero, $-273^\circ$C ($\pm$ the experimental uncertainty). Also compare the slope with Equation (4) above.

**Part II: Heat Engine**

1. Set up the heat engine apparatus as shown in Figure 12.2. Connect the chamber to the piston-and-cylinder unit with tubing. The pressure sensor should also be connected to these. As shown in the figure, the piston-and-cylinder should be attached to a stand with the rotary motion sensor above it. There is a tray attached to the piston. A string should be attached to this tray and looped over the large pulley of the rotary motion sensor and then to a hanging 20 g mass. This string will cause motion of the piston to rotate the rotary motion sensor, while the 20 g mass keeps the string taut. Since the 20 g mass is kept in place throughout the cycle (as is the mass of the piston itself), the energy required to raise and lower it will be positive during part of the cycle and negative during the other part and will thus result in zero net work.

2. Connect the wires to the Pressure Sensor and the Rotary Motion Sensor to the Interface. Double-click the `Rotary Motion Sensor` icon and under measurements, select `Position (m)`. This is the only data needed from this sensor.

3. Set up graph of Pressure vs. Position. Open the `Graph Display` from the list of Displays. Drag the `Angular Position` sensor from the data list to the $x$-axis of the graph. Do the same thing for the `Pressure Sensor` and drag it to the $y$-axis.

4. Do the following steps to collect data for this experiment:
   
   A. First put the chamber into the cold bath; wait for the piston to sink. This moves the system to Point “a” in Figure 12.1. Press the `Start` button in Data Studio to begin taking data.

   B. Place an additional 200g mass on top of the piston tray to increase the pressure. This moves you to Point “b”.

   C. Leaving the 200 g mass in place, move the chamber to the hot bath. This moves you to Point “c”.

   D. Remove the 200 g mass to move to Point “d”.

   E. Complete the cycle by returning the chamber to the cold bath to get to Point “a”.

   F. Complete the cycle by returning the chamber to the cold bath to get to Point “a”.
F. Press the **Stop** button.

5. Does your graph resemble Figure 12.3? The area enclosed in a $PV$ diagram is the work done by the heat engine in going around a cycle. The graph plots $P$ versus $\Delta h$. How can we convert $\Delta h$ to $\Delta V$? Check with your instructor if you are not sure. Use this to calculate the work done by the heat engine in the cycle. Remember to include the negative work done when the volume contracts under constant pressure (chamber transfer hot to cold bath) this negative value is added to the positive value when the piston expands under constant pressure (chamber transfer from cold to hot bath.) The sum of the negative and positive values give the net thermodynamic work. That is:

$$W_{\text{thermodynamic}} = P_1 \Delta V_1 + P_2 \Delta V_2 = P_1 \Delta x_1 A + P_2 \Delta x_2 A,$$  \hspace{1cm} (5)

where $\Delta x_2$ is negative.

6. Once you have calculated the thermodynamic work from the graph, compare this value to the mechanical work you obtain when using the equation

$$W_{\text{mechanical}} = mg\Delta h.$$  \hspace{1cm} (6)

Are they equal? What do you suspect to be the cause of most of the difference?