12 Ideal Gas Law and Heat Engine

Introduction
In this lab, you will explore the Ideal Gas Laws. In Part I, you will observe the proportional relationship between temperature and pressure, creating a graph from data points and using it to infer the proportionality constant, as well as a theoretical temperature for absolute zero. In Part II, you will use a heat engine apparatus to explore the relationship between heat, pressure and volume. You will also observe a way in which heat can be converted to work energy and how much work is done at constant pressure.

Materials
Part I: Data Studio, temperature sensor, pressure sensor, gas chamber, coffee pot, cold water/ice, paint stirrer
Part II: Data Studio, rotary motion sensor, pressure sensor, heat engine, cold water/ice bath, hot water bath

Reference
Giancoli, Physics 6th Edition: Chapter 13, sections: 1,2,4,6,7,9,10; Chapter 15, sections: 1,2,5

Theory
Part I: The ideal gas law relates the pressure, P, volume, V, and temperature, T, in Kelvin of an ideal gas

\[ PV = nRT \]  

1

n is the number of moles and R is the Ideal Gas Constant (equal to 8.31 J/mol*K). In this lab you will hold the volume constant, vary the temperature and study the response of the pressure. We can rewrite the equation as

\[ P = \frac{nR}{V} T = CT \]  

2

where C=(nR/V) and thus the equation you will verify is

\[ P = CT \]  

3

We thus expect a linear graph that passes through the origin if the temperature units are Kelvin. In this lab you will plot the graph in Celsius units and extrapolate the straight line
fit to find where it intersects the x-axis to determine absolute zero. Absolute zero is the
temperature at which the pressure of the gas extrapolates to zero, meaning that the
molecules have the lowest kinetic energy possible.

Part II: The heat engine cycle is a repeatable, closed thermodynamic cycle that is often
represented by a pressure-volume diagram shown in Figure 12.1 below:

![Figure 12.1: Heat Engine Cycle](image)

The cycle has four transitions. In the horizontal segments, $d \rightarrow a$ and $b \rightarrow c$, volume
changes due to changes in temperature while pressure remains constant. During the other
two steps, $a \rightarrow b$ and $c \rightarrow d$, volume changes due to changes in the external pressure on
the piston (the temperature is essentially constant). The heat engine used in this
experiment is a gas confined to a chamber with a moveable piston. The work performed
by the engine lifts a mass against the force of gravity. When a heat engine at a constant
pressure $P$, changes volume by $\Delta V$, the work done by the engine is $W = P\Delta V$. Work is
done when the volume changes and pressure is held constant. If the volume increases, the
work done by the heat engine is positive and when the volume decreases the work done
by the engine is negative.

To see how the engine will work, review the picture above and the equipment at your
table (shown in figure 12.2 below). You will start by placing the chamber connected to
the piston into a cold water bath. This will decrease the volume and lower the
temperature but the pressure will remain essentially constant because the piston will
adjust. This will establish point a.
Next, you will place a mass on the cylinder, this will cause the pressure to increase and
the volume to decrease. This will bring you to point b.
With the mass still on the cylinder, you will place the chamber in a hot water bath. The
volume will expand, raising the piston. The temperature has increased, but the pressure is
held essentially constant by the movement of the piston. This brings you to point c.
Finally, you will remove the mass from the cylinder. This will decrease the pressure and
allow the volume to expand, establishing point d.
To return the beginning of the cycle, you will return the chamber to the cold water bath. The volume and the temperature will decrease, returning to point a.

Since we have a change in the height of the mass when the volume changes, work is done to raise the mass system during the experiment. The change in volume is directly proportional to the change in position (\(\Delta x\)) as measured by the motion sensor. We can measure the net work done over the cycle by measuring the area of the enclosed parallelogram. To find this area, we must find the area under each of the horizontal segments and subtract the area under the lower segment from the area under the upper. In essence, we add the negative work done when the volume decreases from \(d \rightarrow a\) (transfer from hot to cold bath) to the positive work done when volume increases from \(b \rightarrow c\) (transfer from cold to hot bath). For each transition, \(P\Delta V = W\), and therefore, \(W_{\text{net}} = P_1\Delta V_1 + P_2\Delta V_2\). Because the quantity \(\Delta V_2\) in this case is negative, we find the difference between the two work values. The work done by raising the mass is also equal to the change in potential energy, \(mg\Delta h\), where \(\Delta h\) is the distance the piston has moved.

**Procedure**

![Figure 12.2](image-url)
**Part I: Temperature and Pressure**

1. Set up the absolute pressure and temperature sensor in Data Studio. Make sure that the frequency of data acquisition is the same for both sensors, a value of 1 Hz is adequate for the experiment.

2. The gas chamber consists of a temperature sensor and metal chamber to hold the air attached to a paint stirrer. Connect the chamber directly to the pressure sensor.

3. Set up a graph to record changes in temperature and pressure in the chamber. Graph the data with the pressure on the vertical axis and temperature on the horizontal.

4. Add hot water at about 60°C to the thermal jug and then insert the gas chamber assembly.

5. Press the start button in Data Studio.

6. Highlight the best data and do a linear fit. Determine the x-intercept and estimate its uncertainty. The x-intercept is absolute zero. The slope is equal to $\frac{nR}{V}$ (see equation 2) but that is not of interest.

8. Is the x-intercept close to the accepted value of absolute zero within uncertainties, -273°C? If not, why do you think the value was not properly determined?

**Part II: Heat Engine**

1. Connect the Pressure Sensor and the Rotary Motion Sensor to the Interface. Double-click the **Rotary Motion Sensor** icon and under measurements, select **Position** (m). This is the only data needed from this sensor.

2. Set up graph of Pressure vs. Position. Open the **Graph Display** from the list of Displays. Drag the **Angular Position** sensor from the data list to the x-axis of the graph. Do the same thing for the **Pressure Sensor** and drag it to the y-axis.

3. Press the **Start** button in data studio to begin taking data.

   A. First put the chamber into the cold bath; wait for the piston to sink. (This establishes point an in figure 12.1)

   B. Place a 200g mass on top of the heat engine to increase the pressure. (This moves you to b)
C. Leaving the mass on top of the heat engine, move the chamber to the hot bath. (This moves you to c)

D. Remove the 200g mass from the piston to lower the pressure. (Point d)

E. Complete the cycle by returning the chamber to the cold bath. (Point a)

4. Press the Stop button and you should have a graph similar to the following:

![Figure 12.3]

5. Does your graph resemble Figure 12.3? The area enclosed in a PV diagram is the work done by the heat engine in going around a cycle. The graph in Figure 12.3 is P vs. $\Delta h$ but we can convert $\Delta h$ to $\Delta V$ by multiplying by the area of the piston ($\pi r^2$) the piston radius is printed on it. Use this to calculate the work done by the heat engine in the cycle. Remember to include the negative work done when the volume contracts under constant pressure (chamber transfer hot to cold bath) this negative value is added to the positive value when the piston expands under constant pressure (chamber transfer from cold to hot bath.) The sum of the negative and positive values give the net work.

6. Once you have the area, compare the value for the net work from the graph to the theoretical value you obtain when using the equation $W = mg\Delta h$. Can we say the net thermodynamic work is realized in the movement of the mass? If not, why do you think these values are not equal?
For your Lab Report
Include a sample calculation of work as calculated by $\Delta P \Delta V$ with your estimates for the uncertainty of the piston radius, the position and pressure measurements. Also give a sample calculation of $W = mg \Delta h$ using estimates the uncertainty of the height and mass measurements. Compare these values to see if they are equal within the propagated uncertainties.