RC Decay

Introduction:

A capacitor is a device for storing charge and energy. It consists of two conductors insulated from each other. A typical capacitor is called a parallel-plate capacitor and is symbolized \(-\mid\,-\mid\,-\mid\) or \(-\mid\,-\mid\). It consists of two conduction plates separated by a small distance. When the two plates are connected to the two wires of a charging device, e.g., a battery, positive charge is transferred to the plate connected to the positive terminal of the charging device and an equal magnitude of negative charge is transferred to the other plate. This flow of charge continues until the potential difference between the conductors equals the applied potential difference, e.g., the potential difference between the battery terminals. The amount of charge \((Q)\) depends on the geometry of the capacitor and is directly proportional to the potential difference \((V)\). The proportionality constant is called the capacitance \((C)\).

\[
Q = C \cdot V
\]

The gap between plates is usually filled with a dielectric material to increase the capacitance. Charge is measured in coulombs and voltage in volts, so capacitance has units of coulombs/volt that is called a farad or F.

This experiment will investigate the discharge of a capacitor through a resistance. The increase or decrease of charge on the capacitor is exponential in character (as a function of time in this case). This is an important type of behavior in nature since it occurs whenever the instantaneous change in a quantity is only dependent on the amount present at that time. Population and bacterial growth, money in a savings account, and energy use may follow such a law.

Written in electrical symbols the charge, as a function of time, on a discharging capacitor is given by:

\[
Q = Q_0 e^{-t/(RC)} \tag{6.1}
\]

where

\(Q\) is the instantaneous charge at time \(t\),
\(Q_0\) is the initial charge at the beginning of the time interval \(t\),
\(R\) is the resistance in the circuit,
\(C\) is the capacitance of the capacitor used, and
\(e\) is the base of the nature logarithm, \(e = 2.71828\ldots\).

The product \(RC\) is called the time constant of the circuit. It has units of ohms-farads, (which also equals seconds), and is the time required for the charge on the capacitor \(Q\) to fall from \(Q_0\) to \(Q_0/e\), as can be seen by substituting time, \(t = RC\) into Equation 6.1 above. The value \(1/e \approx 0.37\), so at \(t = RC\), \(Q = 0.37 Q_0\). Since this contains an awkward, rounded constant value, \(t_{1/2}\) or half time is usually determined experimentally instead of \(RC\) time directly. \(t_{1/2}\) is defined as the time necessary for the
Q to increase or decrease to \( \frac{1}{2} \) of its maximum value. When the charge has decayed to \( \frac{1}{2} \) of the original charge, we have \( Q = \frac{1}{2} Q_0 \) which can be solved to give \( \frac{Q}{Q_0} = \frac{1}{2} \).

Replacing \( Q \) with the right side of Equation 6.1, we have:

\[
\frac{1}{2} = e^{-t/\tau} = \exp\left(-\frac{t_{1/2}}{RC}\right)
\]

where \( \exp(n) \) stands for \( e \) to the power \( n \). Taking the ln of both sides (the inverse operation as \( \exp(n) \) ) to “cancel out” the \( \exp(n) \) operation, we have:

\[
\ln \frac{1}{2} = -\frac{t_{1/2}}{RC}
\]

Rearranging this yields:

\[
t_{1/2} = 0.693RC
\]

Figure 6.1: Circuit used to measure slow discharge of a capacitor. \( R_1 = 10M\Omega \), \( C = 1.0\mu F \), \( R_v = 10M\Omega \).

**Procedure: Slow exponential decay**

The circuit shown in Figure 6.1 will be used to measure slow discharge of a capacitor. The voltmeter here is connected in series with the other components. This is an unusual use of a voltmeter (voltmeter are normally placed in parallel to other components). Figure 6.1 shows the voltmeter in terms of its internal resistance. There are obviously many more components inside the voltmeter to allow it to function as a
voltmeter, but these are not shown here so we can focus on the effect the voltmeter has on the flow of current through the circuit. Momentarily touching the positive wire from the power supply to the indicated node charges the capacitor. When the power supply is then disconnected, the capacitor will discharge through the resistor and voltmeter. (Note that it is important that the power supply wire be physically disconnected after charging in order for this experiment to function properly.) The size of the current, however, is extremely small and would require a very sensitive meter to record. However, the large resistance of a multimeter used in the voltmeter mode results in a measurable voltage drop with even a small current flow. Using the voltmeter in series thus allows for a reasonable measure of the current change, and therefore the charge change, with time, since the current is proportional to the voltage according to Ohm's Law.

Using a stopwatch and working in teams, readings from the voltmeter should be taken every 2 seconds after the capacitor is charged. (One person watches the time and calls out intervals, one reads the voltmeter out loud; and the third records the voltage.) Graph the voltage vs. time. We now have three ways to determine the time constant:

1. From the circuit components $R = R_1 + R_v$ and $C$ to give $RC$. It is important to remember that the resistance $R$ includes the resistance of the voltmeter $R_v$ since it is in series with $R_1$.

2. By picking the half time off our graph and using Equation 6.2 to calculate $RC$.

3. By using the Excel trendline (display the equation) with an exponential fit. Compare the $RC$ values (the time constants) from these three methods using the uncertainties in each. Which method is most accurate? Also re-plot the data using a logarithmic axis for voltage with trendline. This should produce a straight line plot.

There will be an assortment of other resistors and capacitors available to you for further exploration.