

Motivation for Special Relativity

Coincidences in classical physics led Einstein to propose SR.

Classical mechanics: motion of things (in space, as function of time)

Space: 3-dimensional and Euclidean => Cartesian coords can be established and distance between 2 points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $d = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}$

Absolute space defines a standard of rest.

Time exists independently of space and is 1-dimensional.

Space and time are homogeneous and space is isotropic.

An **event** is a particular point in space at a particular time: (x, y, z, t)

Motion of a particle consists of a series of events.

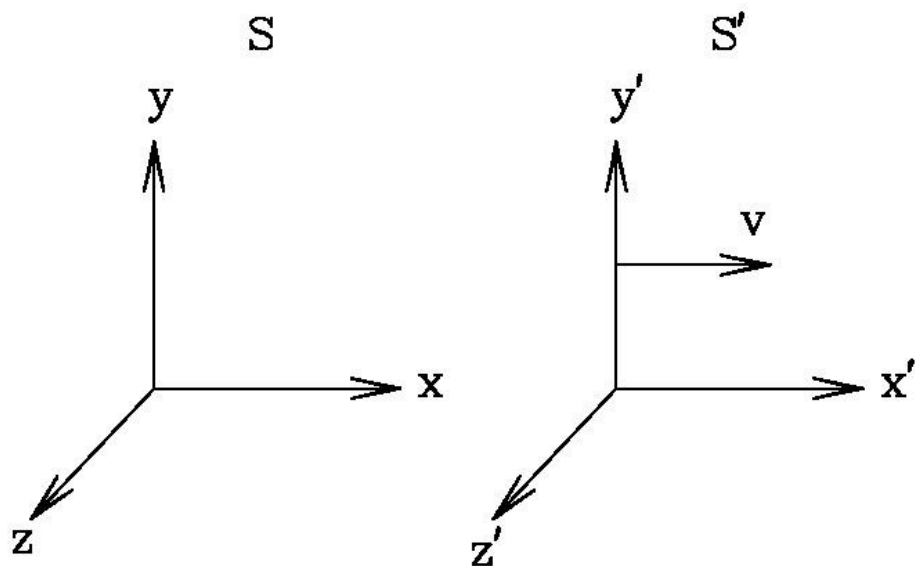
Newton's Laws of Motion:

1. Free particles (those on which no forces act) move with constant velocity (i.e., with constant speed along a straight line).
2. $\mathbf{f} = m \mathbf{a}$
3. $\mathbf{f}_{12} = -\mathbf{f}_{21}$

Newton's 1st Law applies in the ref frame of absolute space and also in any frame that moves at constant velocity wrt absolute space; such frames are called **inertial reference frames**.

Technical note: For each ref frame, there are an infinite number of ref coord systems. All are equally valid because of homogeneity and isotropy.

2 inertial ref frames S and S' in “standard configuration”:



S' moves with speed v along x -axis rel to S.

$t = t' = 0$ when origins coincide

Suppose an event has coords (x, y, z, t) rel to S and (x', y', z', t') rel to S'

Standard Galilean transformation:

$$x' = x - vt \quad y' = y \quad z' = z \quad t' = t$$

Non-standard config: $x' = x - vt$ $t' = t$

Suppose a particle moves with velocity \mathbf{u} rel to S and \mathbf{u}' rel to S' :

$$\mathbf{u}' = \mathbf{u} - \mathbf{v}$$

Acceleration: $\mathbf{a}' = d\mathbf{u}'/dt' = d(\mathbf{u} - \mathbf{v})/dt = d\mathbf{u}/dt = \mathbf{a}$

(invariance of accel under transformations between IFs)

Mass and force are assumed to be invariant

=> invariance of Newton's 3 Laws (“Galilean relativity”)

=> no mechanical experiment can indicate which inertial frame is at rest wrt absolute space!

Most classical physicists: This is an interesting coincidence.

Einstein: This suggests that absolute space is a dubious concept.

Side note: Galilean relativity is a good thing for us, since we're hurtling through space but could still work out the laws of mechanics!

Classical electrodynamics:

Maxwell's equations tell us how the fields are established by the charges and currents, and the **Lorentz force law** tells us the force on a charged particle, given the fields at its location:

SI (mks) units

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Gaussian (cgs) units

$$\nabla \cdot \mathbf{E} = 4\pi \rho$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{F} = q \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right)$$

See Appendix in Griffiths regarding unit conversions.

Refresher on derivatives in vector calculus (sec 1.2 of Griffiths):

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

“del” operator

$$\begin{aligned} \nabla \cdot \mathbf{A} &= \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (A_x, A_y, A_z) \\ &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \end{aligned}$$

divergence (del dotted into a vector)

$$\nabla A = \left(\frac{\partial A}{\partial x}, \frac{\partial A}{\partial y}, \frac{\partial A}{\partial z} \right)$$

gradient (del times a scalar)

$$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

Laplacian (divergence of gradient)

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

curl (del crossed into a vector)

$$\nabla^2 \mathbf{A} = (\nabla^2 A_x, \nabla^2 A_y, \nabla^2 A_z)$$

$$\nabla \times (\nabla A) = 0$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

This is often taken to define $\nabla^2 \mathbf{A}$, rather than the above appeal to Cartesian coords

Taking the divergence of the Ampere-Maxwell eqn yields:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad (\text{the continuity eqn; conservation of electric charge})$$

Does Galilean relativity apply to electrodynamics?

Apparently not: e.g., Lorentz force law depends on particle's velocity.

But: Perhaps \mathbf{E} and \mathbf{B} look different in different IFs.

Simple demonstration that E&M does not exhibit Gal rel: **em waves**

Maxwell's eqn's with
no charges or currents:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0 & \nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 & \nabla \times \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \end{aligned}$$

$$\begin{aligned} \nabla \times (\nabla \times \mathbf{E}) &= \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \\ &= -\frac{1}{c} \frac{\partial}{\partial t} (\nabla \times \mathbf{B}) \\ &= -\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \end{aligned}$$

$$\Rightarrow \nabla^2 \mathbf{E} = \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

This is the wave eqn!

Similarly:
$$\nabla^2 \mathbf{B} = \frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Consider a scalar field and only x -dependence:

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \quad \text{Try sol'n } f(x, t) = g(z \equiv x - vt) ; \text{ a wave w/ speed } v$$

$$\frac{\partial f}{\partial x} = \frac{dg}{dz} \frac{\partial z}{\partial x} = \frac{dg}{dz}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{dg}{dz} \right) = \frac{d^2 g}{dz^2} \frac{\partial z}{\partial x} = \frac{d^2 g}{dz^2}$$

$$\frac{\partial f}{\partial t} = \frac{dg}{dz} \frac{\partial z}{\partial t} = -v \frac{dg}{dz}$$

$$\frac{\partial^2 f}{\partial t^2} = -v \frac{\partial}{\partial t} \left(\frac{dg}{dz} \right) = -v \frac{d^2 g}{dz^2} \frac{\partial z}{\partial t} = v^2 \frac{d^2 g}{dz^2}$$

$$\Rightarrow \frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

If Galilean relativity applied to Maxwell's eqns, then:

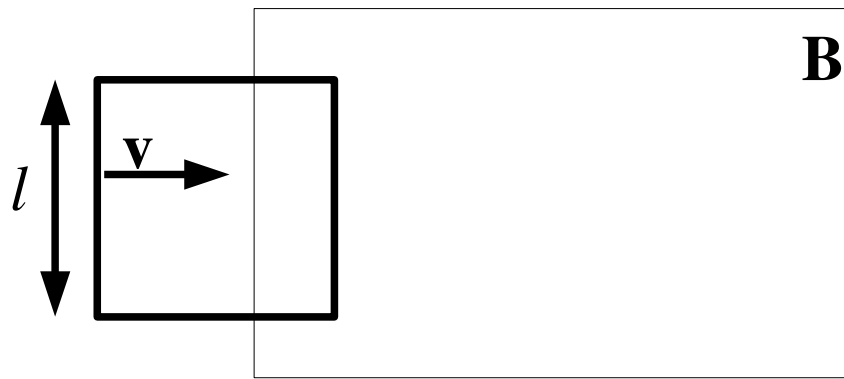
1. Form of Maxwell's eqns must be the same in all IFs
=> observers in all IFs measure the same speed c for em waves.
2. Observers in different IFs should measure different values for the speed of an em wave, depending on their relative velocity.

1 and 2 are inconsistent => Maxwell's eqns CANNOT respect Galilean relativity!

Note: The wave eqn can be written: $\square \mathbf{E} = 0$; $\square \mathbf{B} = 0$

$\square \equiv \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2$ is called the d'Alembertian operator.

A suggestive coincidence: wire loop moving through a stationary \mathbf{B}



\mathbf{B} (out of page)

Magnetic force on charge q :
 $F = qvB/c$ (down)

emf $\mathcal{E} = \oint (\mathbf{F}/q) \cdot d\mathbf{l} = -vBl/c$ (minus sign since dir. of $\mathbf{B} \Rightarrow$
 counterclockwise integration)

Also note: $d\Phi/dt = B dA/dt = Blv \Rightarrow \mathcal{E} = -c^{-1} d\Phi/dt$

What would an observer moving with the loop see?

Changing flux \Rightarrow emf by Faraday's Law: $\nabla \times \mathbf{E} = -c^{-1} \partial \mathbf{B} / \partial t$

$$\int (\nabla \times \mathbf{E}) \cdot d\mathbf{A} = \oint \mathbf{E} \cdot d\mathbf{l} = \mathcal{E} \quad (\text{using Stokes' theorem and } \mathbf{F} = q\mathbf{E})$$

$$= -c^{-1} d(\int \mathbf{B} \cdot d\mathbf{A}) / dt = -c^{-1} d\Phi/dt$$

$\Rightarrow \mathcal{E} = -c^{-1} d\Phi/dt$ Moving observer gets it right. **Dumb luck?**

Three possibilities were apparent to Einstein:

1. Status quo: There is an absolute standard of rest. Galilean rel. in mechanics is just a coincidence. The “relativity” coincidences in electrodynamics are nothing deep.
2. There is no absolute standard of rest. Galilean rel. applies to all of physics. Maxwell's eqns are wrong. Seek new laws of electrodynamics consistent with Gal. rel.
3. There is no absolute standard of rest. There exists a relativity principle that applies to all of physics, but it's not Gal. rel. Laws of mechanics must be revised. Perhaps laws of electrodynamics can survive intact.

Einstein chose 3: introduced a new type of relativity, SR, in which observers in all IFs measure the same speed of light, regardless of the source's motion rel. to the frame.

His decision was supported by the **Michelson-Morley experiment**.