Lecture #7

- Review last class: Sound Waves

- Discussion of Question for Class #2

**MHD Waves**

Propagation of small-amplitude, homogeneous, ideal MHD

Equations: Faraday's law \[ \frac{\partial B}{\partial t} = \nabla \times \mathbf{U} \times \mathbf{B} \]

Mass Continuity Equation

Momentum equation

Equation of State: Adiabatic eq. of state

\[ \mathbf{U} = \mathbf{U}_1 \rightarrow \text{time-independent; uniform} \]

\[ \rho_m = \rho_0 + \rho_m \]

\[ \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1 \]

\[ p = p_0 + p_1 \]

so... linearized first-order equations

\[ \frac{\partial (\rho_0 + \rho_m)}{\partial t} + \nabla \cdot \left[ (\rho_0 + \rho_m) \mathbf{U}_1 \right] = 0 \]

\[ \frac{\partial \rho_m}{\partial t} + \rho_m \nabla \cdot \mathbf{U}_1 = 0 \]

\[ \begin{align*} 
\left( \rho_0 + \rho_m \right) \frac{d\mathbf{U}_1}{dt} &= -\nabla (p_0 + p_1) + \frac{1}{\rho_0} \left[ \mathbf{B}_0 + \mathbf{B}_1 \right] \times (\mathbf{B}_0 + \mathbf{B}_1) \\
\rho_0 \frac{d\mathbf{U}_1}{dt} &= -\nabla p_1 + \frac{1}{\rho_0} (\nabla \times \mathbf{B}_1) \times \mathbf{B}_0 
\end{align*} \]
\[ \frac{\partial (B_0 + B_1)}{\partial t} = \gamma \times \left[ (U_0 + U_1) \times (B_0 + B_1) \right] \]

\[ \frac{\partial B_1}{\partial t} = \gamma \times (U_1 \times B_0) \]

\[ \frac{d}{dt} \left[ (B_0 + B_1) \left( \rho_{m_0} + \rho_{m_1} \right) \right] = 0 \]

\[ \frac{d}{dt} \left( \frac{p_1}{\rho_{m_0}} \right) = 0 \]

\[ p_1 = \gamma \left( \frac{p_0}{\rho_0} \right) \rho_1 \]

**Speed of sound:**

\[ \frac{v_s^2}{\gamma} = \gamma \frac{p_0}{\rho_{m_0}} = \gamma \frac{k_B T_0}{m} \]

Fourier analysis, i.e.: \( \tilde{U} \rightarrow i \tilde{k} \); \( \partial / \partial t \rightarrow -i \omega \)

So...

\[ -i \omega \tilde{\rho}_m + i \rho_{m_0} \tilde{k} \cdot \tilde{U} = 0 \]

(I)

\[ -i \rho_{m_0} \omega \tilde{U} = i \left( \frac{k^2}{\mu_0} \right) \times \tilde{B}_0 - i k \tilde{\rho} \]

(II)

\[ -i \omega \tilde{B} = i k \times (\tilde{U} \times \tilde{B}_0) \]

(III)

\[ \tilde{p} = \frac{v_s^2}{\gamma} \tilde{\rho}_m \]

(IV)

\[ \tilde{B} = U_1 \]

\[ \tilde{B} = \tilde{B}_1 \]

\[ p = p_1 \]
\[ \dot{P} = V_s^2 \rho_m = \frac{\rho_{mo} k^2 \cdot \vec{U}}{\omega} \]

\[
\dot{P} = \frac{\rho_{mo} V_s^2 k^2 \cdot \vec{U}}{\omega}
\]

(II) \text{ Eq. (II)}

\[
-i \omega \rho_{mo} \dot{\vec{U}} = \frac{i}{\mu_0} (k^2 \hat{\vec{B}}) \times B_0 - i k \frac{\rho_{mo} V_s^2}{\mu_0} k \cdot \vec{U}
\]

\[
x i \omega \frac{\rho_{mo}}{\mu_0} \dot{\vec{U}} = -\frac{\omega}{\mu_0 \rho_{mo}} (k^2 \hat{\vec{B}}) \times B_0 + V_s^2 k \cdot \vec{U}
\]

Using Eq. (III) for \( \vec{b} \) we get

\[
\omega \dot{U} = \frac{1}{\mu_0 \rho_{mo}} \left\{ k^2 (k^2 \times [\vec{U} \times \vec{B}_0]) \right\} \times \vec{B}_0 + V_s^2 k \cdot \vec{U}
\]

(III) \text{ Eq. (III)}

Assuming

\[
\vec{B}_0 = B_0 \hat{z}
\]

\[
\vec{k} = k \sin \theta \hat{x} + k \cos \theta \hat{z}
\]

(V) has

\[
\vec{U}_x = \ldots
\]

\[
\vec{U}_y = \ldots
\]

\[
\vec{U}_z = \ldots
\]

\[
\vec{U} \times \vec{B}_0 = \begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
\dot{U}_x & \dot{U}_y & \dot{U}_z \\
0 & 0 & B_0
\end{vmatrix} = \vec{U}_y B_0 \hat{\vec{j}} - \vec{U}_x B_0 \hat{\vec{k}}
\]

\[
k \times \vec{U} \times \vec{B}_0 = \begin{vmatrix}
\hat{x} & \hat{y} & \hat{z} \\
k \times \hat{x} & k \times \hat{y} & k \times \hat{z} \\
k \times \vec{U}_0 \times \vec{B}_0 & 0
\end{vmatrix} = \vec{U}_y B_0 k \times \hat{k} \hat{\vec{j}} - \vec{U}_x B_0 k \times \hat{\vec{i}}
\]

\[
\vec{U}_0 - \vec{U}_0 \times \vec{B}_0 \hat{k} = \vec{U}_y B_0 k \sin \theta \hat{\vec{j}} - \vec{U}_x B_0 k \sin \theta \hat{\vec{i}}
\]

\ldots \text{ etc}
and dividing by $k^2$

\[
\left(\frac{\omega}{k}\right)^2 \begin{bmatrix} \vec{u}_x \\ \vec{u}_y \\ \vec{u}_z \end{bmatrix} = V_A^2 \begin{bmatrix} \vec{u}_x \\ \vec{u}_y \cos^2 \theta \\ 0 \end{bmatrix} + V_S^2 \begin{bmatrix} \vec{u}_x \sin^2 \theta + \vec{u}_z \sin \theta \cos \theta \\ 0 \\ \vec{u}_x \sin \theta \cos \theta + \vec{u}_z \cos \theta \end{bmatrix}
\]  

(\text{VI})

where $V_A = \frac{B_0}{\sqrt{\mu_0 \rho_{w_0}}}$, Alfvén velocity

\[
\frac{\omega}{k} \equiv V_p, \text{ phase velocity}
\]

Eq. (\text{VI}) can be written as

\[
\begin{bmatrix} V_p^2 - V_S^2 \sin^2 \theta - V_A^2 & 0 & -V_S^2 \sin \theta \cos \theta \\ 0 & V_p^2 - V_A^2 \cos^2 \theta & 0 \\ -V_S^2 \sin \theta \cos \theta & 0 & V_p^2 - V_S^2 \cos^2 \theta \end{bmatrix} \begin{bmatrix} \vec{u}_x \\ \vec{u}_y \\ \vec{u}_z \end{bmatrix} = 0
\]

if this is zero $\rightarrow$ non-trivial solutions for $\vec{u}$

\[
D(k, \omega) = (V_p^2 - V_S^2 \sin^2 \theta - V_A^2) (V_p^2 - V_A^2 \cos^2 \theta) (V_p^2 - V_S^2 \cos^2 \theta) + (V_S^2 \sin \theta \cos \theta) (V_p^2 - V_A^2 \cos^2 \theta) (V_S^2 \sin \theta \cos \theta) = 0
\]

(\text{VII})

\[
D(k, \omega) = (V_p^2 - V_A^2 \cos^2 \theta) \left[ (V_p^2 - V_S^2 \sin^2 \theta - V_A^2) (V_p^2 - V_S^2 \cos^2 \theta) - (V_S^2 \sin \theta \cos \theta) (V_S^2 \sin \theta \cos \theta) \right]
\]

$= (V_p^2 - V_A^2 \cos^2 \theta) \left[ V_p^4 - V_p^2 (V_A^2 + V_S^2) + V_A^2 V_S^2 \cos^2 \theta \right] = 0$

(\text{VII})
The equation VII has three roots (show it!)

(A) \( V_p^2 = \frac{1}{2} \left( V_A^2 + V_s^2 \right) - \frac{1}{2} \left[ \left( V_A^2 - V_s^2 \right)^2 + 4 V_A^2 V_s^2 \sin^2 \theta \right]^{1/2} \)

(B) \( V_p^2 = V_A^2 \cos^2 \theta \)

(C) \( V_p^2 = \frac{1}{2} \left( V_A^2 + V_s^2 \right) + \frac{1}{2} \left[ \left( V_A^2 - V_s^2 \right)^2 + 4 V_A^2 V_s^2 \sin^2 \theta \right]^{1/2} \)

A: slow magnetoionic mode

B: transverse Alfvén mode (or shear Alfvén mode)

C: fast magnetoionic mode

\[ V_p \]

\[ \sqrt{V_A^2 + V_s^2} \rightarrow \text{fast magnetoionic} \]

\[ V_A \quad \rightarrow \quad V_s \]

\[ \frac{1}{2} \theta \]

Case \( V_A > V_s \)

\[ V_p \]

\[ \sqrt{V_A^2 + V_s^2} \rightarrow \text{transverse alfvén} \]

\[ V_A \rightarrow V_s \]

\[ \frac{1}{2} \theta \]

Case \( V_A < V_s \)
(B) Transverse (or shear) Alfvén mode

\[ V_p^2 = V_N^2 \cos^2 \theta \]

Substituting in Eq. (V5)

\[
\begin{bmatrix}
V_N^2 \cos^2 \theta - V_S^2 \sin^2 \theta - V_A^2 \\
0 \\
-V_S^2 \sin \theta \cos \theta
\end{bmatrix}
\begin{bmatrix}
U_x \\
V_y \\
U_z
\end{bmatrix} = 0
\]

\[
(V_N^2 \cos^2 \theta - V_S^2 \sin^2 \theta - V_A^2) U_x - V_S^2 \sin \theta \cos \theta U_z = 0
\]

\[-V_S^2 \sin \theta \cos \theta U_x + (V_A^2 - V_S^2) \cos^2 \theta U_z = 0
\]

\[
\Rightarrow \quad \vec{\ddot{U}} = \vec{\dot{U}_z} = 0 \quad ; \quad \ddot{U}_y \neq 0
\]

\[
-i \omega \vec{B} = i \frac{k}{\epsilon} \times (\vec{\dot{U}} \times \vec{B}_0)
\]

\[
-i \omega \vec{B}_y = i \frac{k}{\epsilon} \vec{U}_y B_0 \cos \theta - \vec{E}_y = -B_0 \frac{\dot{U}_y}{\omega} \frac{k}{\epsilon} \cos \theta
\]

\[
\vec{\ddot{B}} = \vec{0}
\]

\[
\vec{B} = (0, B_y, 0)
\]

So

\[
\vec{B}_y = -B_0 \left( \frac{\ddot{U}_x}{V_A} \right) \text{sign} (\cos \theta)
\]

\[
\vec{E} = -\vec{\ddot{U}} \times \vec{B}_0
\]

\[
\vec{E} = (\vec{\ddot{U}} \times \vec{B}_0)
\]
\[
\begin{aligned}
\frac{d}{dt} \rho_m = \Delta \rho_{m0} \mathbf{v} \cdot \mathbf{\hat{v}} \\
\rho_m = \frac{1}{w} \rho_{m0} \mathbf{k} \cdot \mathbf{\hat{v}} = 0
\end{aligned}
\]

**Eigenvectors for Transverse Alfvén Mode**

Fluid motions are entirely \textbf{transverse}; no compressional component \(( \mathbf{E} \cdot \mathbf{\hat{v}} = 0 \))

so the fluid pressure \& temperature play no role in the propagation of this mode.

The \textbf{Poynting flux}: \( \mathbf{S} = \left( \frac{1}{\mu_0} \right) \mathbf{E} \times \mathbf{B} \)

\[
= \left( \frac{1}{\mu_0} \right) \mathbf{E}_x \times \mathbf{B}_y = \frac{1}{\mu_0} \mathbf{E}_x \mathbf{B}_y \frac{\partial}{\partial z}
\]

\( \parallel \mathbf{B}_0 \Rightarrow \)

\( \rightarrow \) The electromagnetic energy flow is along the \textbf{magnetic field} (independent of the wave normal angle)

The group velocity:

\[
\begin{align*}
\mathbf{v}_g &= \mathbf{v}_E \cdot \mathbf{\omega} = \frac{\partial \mathbf{E}}{\partial \mathbf{k}} = \mathbf{v}_A \mathbf{z} \\
\mathbf{v}_g &= \frac{\partial \mathbf{E}}{\partial \mathbf{k}} = \mathbf{B}_0 \cdot \mathbf{v}_A \mathbf{\hat{z}} \\
\mathbf{v}_g &= \frac{\partial \mathbf{w}}{\partial \mathbf{k}} = \frac{\mathbf{B}_0}{\sqrt{\mu_0 \eta}} \mathbf{v}_A \mathbf{\hat{z}}
\end{align*}
\]

\( \left( \begin{array}{c}
V_p^2 = V_A^2 \cos^2 \theta \\
\frac{V_p}{k} \end{array} \right) \)

\( \left( \begin{array}{c}
\omega^2 = V_A^2 \cos^2 \theta \\
\mathbf{\omega} = V_A \cos \theta \\
\mathbf{w} = \frac{1}{k} \cdot \mathbf{B}_0 \end{array} \right) \)

\( \left( \begin{array}{c}
\mathbf{w} = \frac{1}{\sqrt{4 \sin^2 \theta}} \end{array} \right) \)
So the group velocity is parallel to the static magnetic field.

**Analogy: Taut String**

Velocity of propagation of a wave along a taut string is

\[ V_p = \sqrt{\frac{T}{\lambda m}} \]

where \( T \): tension
\( \lambda m \): mass per unit length

Magnetic pressure tensor: isochoric pressure + tension force per unit area

\[ \frac{B^2}{\mu_0} \]

so if we

\[ T \leftrightarrow \frac{B^2}{\mu_0} \Delta A \]

\( \Delta A \) unit area

\( \lambda m \leftrightarrow \rho m \Delta A \) mass per unit length

so

\[ V_p = \frac{B}{\sqrt{\mu_0 \rho m}} \]

So propagation of the transverse (or shear) Alfvén waves is analogous to propagation on a taut string with \( B \) providing the tension and \( \rho m \) the mass / unit length.

waves on different magnetic field lines propagate independently as though they were on separate strings.
(A) and (C) Fast and Slow Magnetosonic Modes

\[
V_p^2 = \frac{1}{a} (V_A^2 + V_s^2) - \frac{1}{2} \left[ (V_A^2 - V_s^2)^2 + 4 V_A^2 V_s^2 \sin^2 \theta \right]^{1/2} \quad (A)
\]

\[
V_p^2 = \ldots \quad \ldots \quad (C)
\]

involve \( V_A \) (magnetic field) \& \( V_s \) (plasma pressure) so it's magnetosonic.

For oblique angles of propagation \( k \cdot u \neq 0 \) so in contrast with transverse Alfvén mode, magnetosonic modes have both longitudinal (compressional) and transverse components.

The dispersion relation is complicated.

First we consider the case \( \theta = 0 \)

so the determinant can be written as

\[
\begin{bmatrix}
V_p^2 - V_A^2 & 0 & 0 \\
0 & V_p^2 - V_s^2 & 0 \\
0 & 0 & V_p^2 - V_A^2
\end{bmatrix}
\begin{bmatrix}
\tilde{u}_x \\
\tilde{u}_y \\
\tilde{u}_z
\end{bmatrix}
= 0
\]

Two roots: \( V_p^2 = V_A^2 \) \& \( V_p^2 = V_s^2 \)

which one is slow \& which one is fast? \( \Rightarrow \) depend on \( V_A \geq V_s \) or \( < V_s \)

If \( V_A > V_s \), \( V_p^2 = V_A^2 \) is the fast \& \( V_p^2 = V_s^2 \) is the slow mode

If \( V_A < V_s \), \( V_p^2 = V_s^2 \) is the fast \& \( V_p^2 = V_A^2 \) is the slow mode

For \( V_p^2 = V_A^2 \) \( \tilde{u}_z = 0 \) \( \tilde{u} = (\tilde{u}_x, 0, 0) \) (taking \( \tilde{u}_y = 0 \))

so \( \tilde{u} \times B_0 = \begin{vmatrix} i & j & k \\ U_x & 0 & 0 \\ 0 & 0 & B_0 \end{vmatrix} = -U_x B_0 k \)

\[
\begin{bmatrix}
\tilde{B}_x \\
\tilde{B}_y \\
\tilde{B}_z
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
U_x B_0 \\
U_y B_0 \\
U_z B_0
\end{bmatrix}
\Rightarrow \tilde{B}_x = -U_x B_0 k \]
\( E = - \ddot{U}_x \ddot{B}_0 \) \quad \Rightarrow \quad \ddot{B} = (\ddot{B}_x, 0, 0) \\
\ddot{E}_y \neq 0 \\
\Rightarrow \quad \ddot{\rho}_m = 0 \quad \text{no density compression} \\
\Rightarrow \text{The eigenvectors for this mode have the properties of a transverse electromagnetic wave. (This only occurs at } \theta = 0) \\
\text{For } V_p^2 = V_s^2 \quad \ddot{U}_x = 0 \quad \Rightarrow \ddot{U} = (0, 0, \ddot{U}_z) \quad \ddot{U} \times B_0 = 0 \\
\Rightarrow \ddot{B} = (0, 0, 0) \quad \ddot{E} = (0, 0, 0) \\
\ddot{\rho}_m \Leftrightarrow k \cdot \ddot{U} \neq 0 \quad \Rightarrow \quad \ddot{B}_z \ddot{U}_z \\
\text{so } \ddot{\rho}_m = \rho_0 \left( \frac{\ddot{U}_z}{V_p} \right) \\
\text{the electromagnetic fields are zero and fluid is density compression. The eigenvectors have the properties of a sound wave.} \\
\text{For } \theta = \frac{\pi}{2} \\
\begin{bmatrix} V_p^2 - V_s^2 - V_A^2 \\ 0 \\ V_p^2 \end{bmatrix} \begin{bmatrix} \ddot{U}_x \\ \ddot{U}_z \end{bmatrix} = 0 \\
\Rightarrow V_p^2 = V_s^2 + V_A^2 \quad \Rightarrow \ddot{U}_x \neq 0 \\
\ddot{U} = (\ddot{U}_x, 0, 0) \quad \dddot{U} \times B_0 = -U_x B_0 \ddot{z} \\
\dddot{B} = (0, 0, \dddot{B}_z) \\
\dddot{E} = -\dddot{U}_x \dddot{B}_0 = -\dddot{E}_y \quad \Rightarrow \dddot{E} = (0, \dddot{E}_y, 0) \quad \dddot{\rho}_m = \rho_0 \left( \dddot{U}_z / V_p \right)
The eigenvectors share features of both electromagnetic & sound wave.

For intermediate wave normal angles, the two magnetoacoustic modes have a longitudinal (i.e., sound waves) & transverse (i.e., electromagnetic) components.

Let's take the limit $V_s^2 \ll V_A^2$ (in intermediate wave angle) (look @ (\texttt{#}))

$$\text{transverse part}$$

$$(V_p^2 - V_A^2)(V_p^2 - V_A^2 \cos^2 \Theta)(V_p^2 - V_s^2 \cos^2 \Theta) \times \ldots$$

magnetoacoustic part is

$$(V_p^2 - V_A^2)(V_p^2 - V_s^2 \cos^2 \Theta) = 0$$

$V_p = V_A$ isotropic root
eigenvectors corresponding to a nearly transverse electromagnetic wave with a small longitudinal (i.e., sound wave) component.

$V_p^2 = V_s^2 \cos^2 \Theta$

has eigenvectors corresponding to a sound wave with only a small electromagnetic component $\vec{E} \approx 0$; $\vec{B} \approx 0$

- fast magnetoacoustic wave
- slow magnetoacoustic wave

The plasma pressure is much smaller than the magnetic pressure when $V_s^2 \ll V_A^2$.

→ the fluid motion for the slow magnetoacoustic is constrained to be nearly parallel to the static $\vec{B}$.
The group velocity & phase velocity are parallel to $\mathbf{B}$.

Motion of particles & energy flow is similar to the motion of gas in a parallel array of pipes! ($B \propto$ pipes).

**Nature of the Fast, Shear & Slow Modes - Some Intuition**

- **Sound Wave**: For $k_\perp = 0$ (in our case $k_x = 0 \land \theta = 0^\circ$)
  
  $\mathbf{B} = 0 \land \mathbf{u} \neq 0$

  $\omega = k_{\parallel} c_0$

  The motion along field leaves them undisturbed.

- **Magnetosonic Waves** in the case of

- **Shear Alfvén Waves**: For $V_p^2 = V_A^2 \cos^2 \theta$ that can be written as

  $V_p \times \mathbf{w} = k_{\parallel} V_A \lor k_\perp = 0$

  parallel propagation $\parallel_\perp$
Magnetosonic Modes (Parallel Motion) 

if $k_\perp = 0$ (in our notation) $k = k_\parallel \Theta = 0^\circ$

$\omega^2/k^2 = v_A^2$ and $v_s^2$ for fast & slow or vice versa (depends on $V_A > v_s$ or $< v_s$)

So, for parallel propagation

So, for parallel motion $\rightarrow$

$\omega = k_\parallel V_A$ shear alhoven wave $(0, \hat{u}_y, 0)$ $\perp$

$\omega = k_\parallel V_A$ or $k_\parallel C_s$ magnetosonic modes.

only transverse motion
no compression

$(\hat{u}_x, 0, 0)$ $\perp$ $(0, 0, \hat{u}_z)$ $\perp$

the two motions, i.e., the directions of polarization are perpendicular.

For perpendicular propagation $\rightarrow \Theta = \frac{\pi}{2}$

shear alhoven wave $\rightarrow$ $\omega \perp \hat{u}_x \Rightarrow \omega = 0$
slow mode $\rightarrow \omega = 0$

Fast mode: $\omega = \sqrt{V_A^2 + C_s^2}$ motion $\perp$ $\hat{u} = (\hat{u}_x, 0, 0)$ polarizaton $\perp$ to $B_0$

force driving is simultaneous compression of both pressure and magnetic field
Motion of slow mode is along \( B_0 \). The two modes have perpendicular polarization vector \( \hat{F} \). The shear Alfven wave is in \( y \).

In general: all three modes (shear / slow / fast) are mutually perpendicular.

Let's re-plot \( V_p \) vs \( \Theta \) on a diagram called

\[ \text{Friedricks diagram} \]

Case \( V_A > V_S \)

\[ (V_A^2 + C_s^2)^{1/2} \]

\[ \Theta = \frac{\pi}{2} \]

\[ C_s \]

\[ V_A \]

\[ \Theta = 0 \] slow \( V = C_s \)

"parallel propagation"

\[ \Theta = \frac{\pi}{2} \] \( V_A \) and \( C_s \)

"perpendicular propagation"

\[ \text{the two slow modes} \]

\[ V_p = 0 \]

\[ \text{fast mode} \]

\[ \sqrt{V_A^2 + C_s^2} \]
For $c_s > v_A$

\[ (v_A^2 + c_s^2)^{1/2} \]

$\theta = \pi/2$

- The two slower make parallel phase velocities = $v_A$
- Fast mode = $c_s$

**MHD Observations**

- Alfvén in 1942 predicted MHD waves
- First observed in laboratory [Lundquist 1949]
  [Wilcox et al. 1960]

**Solar Wind - Large amplitude low-frequency fluctuations**

Belcher et al. (1969) showed that the fluctuations are Alfvén waves:
$B_n, B_t, B_r$ superimposed on $U_r, U_t, U_n$

A running average has been subtracted

As we saw for Shear Alfvén Wave $B_y = v \cdot \hat{B}$
the components of $B$ & $U$ are correlated as expected to shear Alfvén modes.
The almost total absence of fluctuations in zero-order $B_0$ and $N_0$ uniquely identifies the mode of propagation as the **transverse Alfvén** mode.

The in-phase nature of correlation $\rightarrow$ waves are propagating outward from the Sun.

**Sourc** of these waves?

**turbulent motions in the photosphere**

Since $E_+ \rightarrow B_0$, this mode is unaffected by the various collisionless damping processes that can exist in a hot, magnetized plasma.

Thus, once generated, the waves can propagate great distance with little or no attenuation.

(In contrast, when hot plasma effects are considered, the fast and slow magnetosonic modes always have a small component of $E$ along the shock $B$ which lead to damping by thermal plasma.)

**Jupiter & Io**

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**Bragg (1964)** - discover that Io control the radio emissions from Jupiter from 10-40 MHz.

**Goldreich & Lynden-Bell (1969)** proposed that Io is a conductor and
and Io's motion drive electromagnetic force that excite transverse Alven waves propagating northward & southward.

Because the propagating wave velocity the Alven wave pattern is swept backward relative to the incident plasma flow forming a v-shaped standing wave.

The electrical currents associated with these Alven waves creates two quasi-field aligned current loops that link Io to Jupiter.

Magnetic field measurements by Voyager 1 during a close flyby of Io confirmed these Alven waves! [Ness et al. 1979]

Io has active volcanos that terr: gases from these volcanos provide electrically conducting atmosphere around Io.

HST (Hubble Space Telescope) also detected auroral light emission from points where the northward & southward propagating Alven wave interact with Jupiter's atmosphere. [Connerney et al. 1993].