Efficiency and uniformity considerations in optical coherent transient devices

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The efficiency and uniformity of photon echoes are investigated in optical coherent transient sequences using temporally overlapped linear frequency-chirped programming pulses. Distortions in the power spectrum of the programming pulses due to edge effects are found to cause fluctuations in echo efficiency as a function of echo time delay. Smoothing the edges of the programming pulse envelopes is found to significantly reduce distortion in the power spectrum of the pulses, which leads to echoes that are both more efficient and more uniform than those generated by pulses without smoothed edges. The effect of programming pulse strength on echo efficiency and uniformity is shown and discussed through simulations for an optically thin medium and through experiment in Tm$^{3+}$:YAG at 4 K. © 2006 Optical Society of America

1. INTRODUCTION

Optical coherent transient (OCT) technology is currently being developed for many rf signal processing applications, including optical pulse shaping and arbitrary waveform generation (AWG),1–3 as well as true-time delay (TTD) for phased array radar.4,5 OCT systems use rare-earth ion-doped crystals to store the interference of optical programming pulses in spectral population gratings, creating a spectral filter by which subsequent input pulses are processed. The processing bandwidth and time–bandwidth product of OCT systems are set by the inhomogeneous and homogeneous linewidths of the optical transition. A bandwidth greater than 100 GHz and time–bandwidth products greater than 10$^5$ are readily available. Development of OCT rf signal processing into practical devices requires a detailed understanding of the efficiency of the OCT process as well as the uniformity of the efficiency over the temporal programming range of the device.

The preferred method for programming an OCT material to perform TTD and AWG involves temporally overlapped linear frequency chirps (TOLFCs),6 where two pulses with linear frequency modulation are overlapped in time. For TTD, the overlapped pulses chirp at the same rate, but with a frequency offset $\Delta$ between them. For AWG, the chirp rates may differ and multiple offset frequencies may be introduced. For simplicity, we will focus on the simpler TTD case in this paper. If the frequency offset between the programming pulses is small compared to the overall bandwidth of the chirp, then the frequency offset is equivalent to a time delay $\tau_D = \Delta / \kappa$ between the pulses, where $\kappa$ is the chirp rate. When the overlapped programming pulses illuminate the OCT material, rare-earth ions at a given frequency within the bandwidth of the overlapped chirps will thus be excited sequentially by the two pulses, with time delay $\tau_D$ between excitations. The delay causes a phase shift between the pulses in the frequency domain, which results in the frequency-dependent population redistribution of ions, such that a spectral population grating is created in the crystal with grating period $1/\tau_D$. This spectral grating introduces a delay $\tau_D$ on all frequency components of an additional input pulse over the bandwidth of the grating. Broadband TTD is critical to the performance of arrayed radar antenna systems. If this spectrally stored information is probed by a single brief pulse, a time-delayed photon echo is produced that is in essence the impulse response of the stored population grating.7 Applications such as those listed above employ one or more such gratings to create a tailored impulse response, resulting in a complex sequence of multiple echoes when probed by a brief pulse. The performance of TTD and AWG devices depends on these echoes having both high efficiency and uniform efficiency over a broad range of programmed time delays.

In this paper we show that the efficiency and uniformity of the photon echoes have strong dependence on the programming pulse strength and envelope shape. The effects of these parameters are investigated first through simulation in the Fourier-transform approximation, where both saturation and propagation effects in the material are ignored. In this case the echoes are dependent only on the spectral interference of the programming pulses, independent of pulse strength. Results of Bloch simulations, which also ignore propagation effects but do take material saturation into account, are also presented. To explore the effects of material saturation on echo efficiency and uniformity, numerical simulations based on the Maxwell–Bloch equations,8,9 which include the effects of propagation of both the input and the output pulses in an optically thick medium, are briefly discussed. Finally,
experimental results for an optically thick crystal that show the dependence of efficiency and uniformity on material saturation and propagation are presented.

2. FOURIER-TRANSFORM APPROXIMATION

A. Theoretical Model

The simplest approach to analyzing OCT processes is known as the Fourier-transform approximation.\(^7\) This method is valid only for weak input pulses (the crystal responds linearly to the pulses) and optically thin media (absorption and propagation effects are ignored). In this approximation the OCT crystal is assumed to respond linearly to the power spectrum of the combined programming pulses, \(E_1(t)\) and \(E_2(t)\). The spectral grating in the crystal is proportional to \(E_1^*(\omega)E_2^*(\omega)\), where \(E_1^*(\omega)\) is the Fourier transform of \(E_1(t)\). The grating then acts as a complex spectral filter that processes further inputs, \(E_3(t)\), as required by the application. In AWG, for example, several chirped programming pulses \(E_1(t)\) and \(E_2(t)\) may be used to program echoes with various amplitudes and time delays, all of which are recalled by a single probe pulse \(E_3(t)\) to produce a high-bandwidth arbitrary waveform. In TTD, after programming \(E_1(t)\) and \(E_2(t)\), the subsequent input \(E_3(t)\) is typically an unknown rf waveform on an optical carrier, which is delayed by the OCT device. Independent of the application, the causal output of the programmed filter is given in the frequency domain as

\[
E_{echo}(\omega) \approx E_1^*(\omega)E_2^*(\omega)E_3(\omega). \tag{1}
\]

For TOLFC programming, pulses 1 and 2 are the overlapped chirped programming pulses, which are frequency offset with respect to one another. Pulse 3 is a modulated waveform on an optical carrier at the center frequency of the chirped pulses. Because the Fourier-transform approximation does not take into account the nonlinear saturation effects that determine the echo efficiency in an actual OCT process, absolute echo efficiencies are not calculated with this method. Echo fluctuation can be examined using this method, however, as long as it is normalized to be independent of overall echo efficiency.

B. Programming Pulses

Echo fluctuations in the Fourier-transform approximation have been investigated for two types of TOLFC envelope, both of which are shown in Fig. 1. Both pulses have rounded edges, which are formed from half-periods of cosine waves that smoothly roll from a center (constant amplitude) section of the pulse to zero amplitude on either side. In both cases, the frequency continues to chirp linearly at the same rate \((\kappa=3.75\ MHz/\mu s)\) over these edges, so that the total chirp bandwidth is the same for both types of chirp. The chirp bandwidth is \(B_C=15\ MHz\), which is limited by the bandwidth of the acousto-optic modulator (AOM) used in the experiment to modulate the frequencies of the pulses. Because the range over which a laser beam can be tuned is typically the limiting factor in practice, the bandwidth defined here is the maximum frequency excursion of the laser beam. Thus there is a loss of bandwidth when smooth edges are introduced on a chirped pulse, as is the case in an actual bandwidth-limited device. The total duration of the chirped pulses (including the smoothed edges) is \(\tau_C=4\ \mu s\). For sharp programming pulse envelopes [shown in Fig. 1(a)], the rounded edges are very brief \((\tau_{edge}=50\ ns)\) and are intended to simulate the rise time of the AOM. In this case, the bandwidth of the edges \((\sim1/\tau_{edge})\) is comparable to the bandwidth of the chirp, \(B_C\).

For smooth envelopes [shown in Fig. 1(b)], each edge makes up 10% \((\tau_{edge}=400\ ns)\) of the total chirp duration. In this case, the edge bandwidth is much smaller than \(B_C\). The smooth edges are not intended to simulate the rise time of the AOM, but rather to reduce oscillations in the frequency domain that result from abrupt edges in a temporal waveform. Such oscillations are commonly referred to as the Gibbs phenomenon\(^{10,16}\) and in general occur in the frequency domain whenever discontinuities in the time domain are present. Theoretically, the ideal method for eliminating such oscillations would be to inverse Fourier transform the desired frequency-domain signal (in this case, a constant amplitude over a given frequency range) and use the resulting temporal waveform (a sine function in this case) to drive the AOM. This method is impractical, however, when applied to complex arbitrary waveforms since it requires temporally long, high-bandwidth, complex programming pulses, which are well beyond current device capabilities. Another possible method for reducing the oscillations is the use of a Hanning window, which reduces unwanted edge effects by multiplying a temporal waveform with a full-period cosine wave, which has a maximum value at the center of the waveform and falls to zero at either edge. This technique has the disadvantage of greatly reducing the useful bandwidth of the chirped pulse, as only ions at the center programming frequency would be fully excited. The approach described in this paper uses the essential features of the Hanning win-
As the time–bandwidth product \((\tau_C B_C)\) of a chirped pulse increases, the power spectrum of the pulse in the frequency domain becomes flatter over the bandwidth of the chirp.\textsuperscript{11} Because of the finite bandwidth and duration of any chirped pulse, however, there will always be some distortion in the power spectrum. In the simulations and experiments presented in this paper, \(\tau_C\) is chosen so as to yield a moderate time–bandwidth product \((\tau_C B_C=60)\) to show the dramatic effect smooth edges have on the distortions in the power spectrum of the chirps, as well as to illustrate the effect of the edges on echo efficiency and uniformity in TTD sequences. In practical devices, longer chirp durations would typically be used to reduce edge effects, although the duration would ultimately be limited by the desired processing time and the upper-state lifetime of the material \((T_1 \approx 800 \mu s\text{ in Tm}^3\text{:YAG})\).

### C. Spectral Overlap of Input Pulses

Because the echo is dependent on the spectral interference of the programming pulses, the bandwidth of ions programmed by the input pulses must be considered. In the frequency domain, the two programming pulses are identical, but are offset by the frequency shift \(\Delta\). Only the ions that have resonant transitions within the frequency range of the overlap of the two chirps contribute to the programmed spectral grating. Because the frequency offset increases linearly with the programmed time delay \(\tau_D\), the number of ions contributing to the grating decreases with increasing delay. For smooth programming pulses, the region of overlap is even narrower due to the loss of excitation bandwidth, which causes efficiency to fall off more quickly as a function of \(\tau_D\). The following expression approximates the full width at half-maximum programming bandwidth for the chirped programming pulses:

\[
B_{\text{prog}} = B_C - \Delta - \tau_{\text{edge}} 2\kappa.
\]  
(2)

The maximum frequency offset used between programming pulses is \(\Delta=5.6\text{ MHz}\), which corresponds to a time delay of \(\tau_D=1.5\ \mu s\). This gives a minimum programming bandwidth of \(B_{\text{prog}}=9.0\text{ MHz}\) for sharp programming pulses and a minimum of \(B_{\text{prog}}=6.4\text{ MHz}\) for smooth programming pulses. The full width at the 1/e point for the field of the brief Gaussian probe pulse in the frequency domain is \(B_{\text{p}}=5\text{ MHz}\) centered on the programmed grating. The duration of the probe pulse is chosen such that its bandwidth is well within the bandwidth of the programmed gratings, so the frequency offsets do not cause the echo efficiency to decrease for longer delays. If the programming bandwidth approaches the probe bandwidth, such that \(B_{\text{prog}}=B_{\text{probe}}\), then a significant portion of the probe pulse’s energy will interact with ions outside of the programming region, leading to distortion in the echo and steady loss in efficiency as \(\Delta\) is increased. The ions have a dephasing time of \(T_2\approx 15\ \mu s\), which also leads to a fall off in echo efficiency as the delay is increased. This dephasing, along with the spectral overlap of the input pulses, determines the upper limit on programmable time delay with the TOLFC method for a given \(B_C\), \(\tau_C\), and \(\tau_{\text{edge}}\). Although increasing \(\tau_{\text{edge}}\) further would reduce distortion in the power spectrum, it would also decrease the programming bandwidth and the range of programmable delays. In OCT applications, chirp parameters must be chosen to provide low levels of fluctuation, while maintaining the condition \(B_{\text{prog}} \approx B_{\text{probe}}\).

### D. Fourier Simulation Results

Even in the weak field limit, the spectral nonuniformity of the programming pulses leads to fluctuation in echo efficiency as a function of \(\tau_D\). Because \(\tau_D\) increases linearly with the frequency offset \(\Delta\) between the programming pulses, the spectral interference of the nonuniform programming pulses is different for each programmed time delay. This delay-dependent interference leads to nonuniform echoes for various delays. Fluctuation is defined as

\[
\text{fluctuation} = \frac{\eta_{\text{rms}}}{\eta_{\text{avg}}} = \frac{\sqrt{\sum_{i=1}^{N} (\eta_i - \eta_{\text{line}})^2}}{\eta_{\text{avg}} \sqrt{N}}, \tag{3}
\]

where \(\eta_i\) is the echo efficiency for time delay \(\tau_D\); \(\eta_{\text{line}}\) is the value of a linear fit to the data at \(\tau_D\); and \(\eta_{\text{avg}}\) is the average echo efficiency over all time delays, which normalizes the data so that fluctuations can be compared independent of absolute efficiency. Echo efficiency \(\eta_i\) is defined as the ratio of the peak echo intensity at \(\tau_D\) to the peak probe intensity. The number of time delays \(N\) used to calculate fluctuation is 14, with echoes uniformly spaced from \(\tau_D=416\text{ ns}\) to \(\tau_D=1500\text{ ns}\), consistent with other simulations and experimental data, which are discussed below. Time delays less than 416 ns are not used because the echoes cannot be clearly resolved from the edge of the probe pulse. Longer time delays are not used because the dephasing of the ions causes an exponential drop in echo intensity as a function of time delay, and we wish to stay in the range where this decay is small and approximately linear. The delay between the programming pulses and the probe pulse is 20 \(\mu s\), which is greater than the dephasing time of the ions but is much less than the upper-state lifetime of the ions. The relative loss in efficiency due to population decay is approximately 1−\(\exp(-2(20\text{ }\mu s/800\text{ }\mu s))\)=0.05.

Figure 2 shows simulated echo intensities for various time delays. With the input parameters given above, the fluctuation [as defined in Eq. (3)] given by the Fourier approximation for sharp chirp envelopes is 0.04 and for smooth chirp envelopes it is 0.01. The sharp edges, which introduce delay-dependent distortion in the Fourier transform of the overlapped chirped pulses, cause significant fluctuation in echo efficiency as a function of delay. The largest fluctuation, which occurs at \(\sim \tau_C/3 \approx 1.3\ \mu s\), is the result of interference between the spectral grating and the distortions in the power spectrum of the chirped programming pulses. Figure 1(a) shows that the distortion in the power spectrum of a chirped pulse with sharp edges has a varying spectral period. When \(\tau_D \approx \tau_C/3\), the frequency offset between the chirps \((\sim B_C/3)\) causes these distortions to overlap constructively, and the spectral period of these combined distortions is \(\sim 3/\tau_C\) at the center.
of the programmed grating. Because the programmed spectral grating has a spectral period of $1/\tau_D \sim 3/\tau_C$, the chirp distortions interfere with the programmed grating near its center. And because the Gaussian probe pulse is centered on the programmed grating, it is the central region of the grating that contributes to the echo. If the distortions are in phase with the grating, a strong echo is produced. If the distortions are out of phase with the grating, a weak echo is produced. This effect leads to a high level of echo fluctuation for time delays $\sim \tau_C/3$ in sequences with sharp programming pulses. For programmed delays other than $\sim \tau_C/3$, the spectral period of the grating is different from that of the distortions in the power spectrum at the center of the grating, so that there is much less interference causing echo fluctuation.

Because this interference occurs only near the center of the grating at the laser frequency, increasing the probe bandwidth, which causes ions with resonant frequencies away from the center frequency to contribute to the echo, leads to decreased fluctuation at $\sim \tau_C/3$ and a more constant level of echo fluctuation for all time delays. As we discussed above, however, the range of programmable delays is limited by the probe bandwidth, making a high-bandwidth probe pulse undesirable. A better approach is to use programming pulses with smoothed edges, which lead to a much lower level of echo fluctuation while causing only a slight loss in programming bandwidth and range of programmable delay. Figure 2 shows that the smooth TOLFC sequences have a much lower fluctuation and there is no apparent increase in fluctuation at $\sim \tau_C/3$.

3. THIN-MEDIUM BLOCH SIMULATION

Numerical simulation based on the Maxwell–Bloch equations in a thin medium constitutes the next level of complexity in OCT analysis. Saturation effects are investigated, but the crystal is assumed to be optically thin ($\alpha L \ll 1$, where $\alpha$ is the absorption coefficient and $L$ is the length of the crystal), so that propagation effects do not need to be considered. The coupling between the electric field and the resonant ions is characterized by the Rabi frequency, $\Omega = p \cdot E / h$, where $p$ is the ionic dipole moment and $E$ is the envelope of the incident field. The peak amplitudes of the chirped programming pulses are both set to the same value, $\Omega_C$. Because the Rabi frequency is defined only in terms of field amplitude, it has no dependence on other chirp parameters, such as chirp rate or chirp bandwidth. When the chirped pulses are overlapped, their combined field drives the ions. Figure 3 shows simulations for various values of $\Omega_C$. The values for chirp bandwidth, chirp duration, probe bandwidth, and time delays are the same as those used in the Fourier simulations. The peak Rabi frequency of the Gaussian probe pulse is $\Omega_p=0.55$ MHz, chosen so that its pulse area, which is defined as the Rabi frequency in radians integrated over time, is equal to $\pi/4$. This pulse area provides a fairly strong echo signal without saturating the medium.

For small values of $\Omega_C$, the fluctuations shown in Fig. 3 are close to the values predicted in the Fourier simulations, as expected for both smooth and sharp TOLFC programming pulses in the weak pulse limit. The fluctuation increases dramatically for both types of programming pulses, however, at large values of $\Omega_C (>0.5$ MHz). This is because the echo efficiency peaks and begins to fall off for large values of $\Omega_C$ due to saturation of the material, while the nonuniformity in efficiency for different delays continues to increase. The amplitude of the delay-dependent distortion in the power spectrum due to edge effects is smaller than the overall grating and so continues to grow approximately linearly with programming pulse strength.

For sharp programming pulses, these thin-medium simulations give both a peak echo efficiency and minimum fluctuation at $\Omega_C=0.50$ MHz. Simulations based on the coupled Maxwell–Bloch equations for an optically thick medium ($\alpha L = 2.0$) were performed with identical in-

![Fig. 2. Echo intensities simulated by the Fourier-transform approximation are shown for various time delays for both sharp envelopes and smooth envelope programming pulse sequences. For sharp programming pulses, the highest level of fluctuation occurs at $\sim \tau_C/3=1.3 \mu s$. In both cases, $\tau_C=4 \mu s$, $B_C=15$ MHz, and $B_{\text{probe}}=5$ MHz.](image1)

![Fig. 3. Simulated values of echo fluctuation with spline interpolations for an optically thin medium. Fluctuation is smaller in sequences with smooth chirped programming pulse envelopes because of the smoother power spectrum of these pulses. The increase in fluctuation for large values of $\Omega_C$ indicates that this fluctuation is related to saturation in the material. Here $B_C = 15$ MHz, $\tau_C=4 \mu s$, $B_{\text{probe}}=5$ MHz, $\Omega_{\text{probe}}=0.55$ MHz.](image2)
put parameters, giving a peak echo efficiency at $\Omega_C = 0.60$ MHz. Greater programming pulse strength is needed for maximum efficiency in this case due to propagation loss caused by absorption in the material. Experimental results discussed in Section 4, however, indicate that this value of $\Omega_C$ leads to relatively high fluctuation. Although optically thick media have the most highly efficient echoes, some of that efficiency must be sacrificed to have highly uniform echoes.

4. EXPERIMENTAL RESULTS

Experiments were performed using the $^{3}H_{4} - ^{3}H_{6}$ transition in 0.1 at. % Tm$^{3+}$:YAG at 4 K with a $\lambda = 793$ nm Ti:sapphire laser, which is frequency stabilized by locking to a spectral hole in the crystal. The laser beam passes through an AOM, which creates the pulses by modulating the beam's amplitude and frequency. A beam splitter is used to direct a small portion of the beam to a photodetector before the crystal, while the rest of the beam is focused into the crystal and imaged onto a second photodetector. In this way the efficiency can be determined by dividing the peak echo intensity after the crystal by the peak probe intensity before the crystal and multiplying by a calibration factor, which is obtained by sending a probe pulse through the crystal at a temperature much greater than 4 K, where there are no resonance losses, and taking the ratio of the probe intensities detected before and after the crystal. The crystal used is relatively optically thick, with $\alpha L = 2.0$, which has been shown in simulations to be a good thickness for high efficiency with strong pulses. All input parameters are the same as those used in the simulations described above, and all data are averaged 60 times. The quoted Rabi frequencies are based on the peak amplitude at the spatial center of the beam.

Figure 4(a) shows experimental echo efficiencies for various time delays (the same delays used in simulations), programmed with sharp programming pulse envelopes. The greatest efficiency occurs for $\Omega_c = 0.60$ MHz, consistent with results from Maxwell–Bloch simulations. The consistent oscillatory behavior of the efficiencies demonstrates that the fluctuations are deterministic saturation effects and not random. Because these fluctuations due to saturation effects are larger than those discussed in the Fourier-transform approximation, major fluctuations are not apparent for $\tau_p - \tau_c/3$. Figure 4(b) shows echo efficiencies for the same set of delays and programming pulse areas, but now with smooth programming pulse envelopes. A slight increase in efficiency is apparent for all programming pulse strengths with smooth chirp envelopes compared with sharp programming pulse sequences. This may be due to the smooth programming pulses having more uniform spectral distributions, which causes less of the echo energy to be spread into the sidebands.

Figure 5 shows experimental echo fluctuations calculated from the data shown in Fig. 4. The minimum fluctuation occurs at $\Omega_c = 0.45$ MHz, showing that there is a trade-off between echo efficiency and uniformity in an optically thick medium. Fluctuations are lower for smooth programming pulse sequences than for sharp programming pulse sequences for all values of $\Omega_C$, consistent with results from thin-medium simulations. This can also be attributed to the lower level of distortion in the power spectrum of these pulses. A higher level of fluctuation for $\Omega_c = 0.25$ MHz is apparent in the experimental results shown in Fig. 5 compared with the simulations shown in...
Fig. 3. This is probably a noise-related effect. Because the echo intensity for \( \Omega_C = 0.25 \text{ MHz} \) is very weak, the signal and noise are at comparable levels. The noise thus contributes significantly to the fluctuations as calculated.

5. CONCLUSION

Efficiency and fluctuation in photon echo sequences have been examined through experiment and simulation. Similarities between experimental results and thin-medium simulations have shown that propagation effects do not contribute significantly to echo fluctuations in a thick medium. The main contributors to nonuniform echo efficiency are programming pulse edge effects and material saturation due to strong programming pulses. Using smooth edges on the programming pulse envelope, although causing a slight loss in programming bandwidth, has a significant benefit of both increasing efficiency and decreasing fluctuation. The fluctuations are greater for large programming pulses because the echo efficiency saturates, while the distortion in the power spectrum continues to grow.

There is ultimately a trade-off between echo efficiency and uniformity. An OCT device that requires a high degree of echo uniformity must be programmed with weaker pulses than a device that requires maximum efficiency at the cost of increased fluctuation, while other devices may be used with intermediate operating conditions as a compromise between these two factors. In general, minimized fluctuation can be obtained with only a slight sacrifice in efficiency. Smoothed edges on programming pulse envelopes have the effect of increasing overall efficiency and lowering fluctuations for all values of \( \Omega_C \), although they do cause a greater falloff in efficiency for longer delays. The choice of programming pulse strength and shape ultimately depends on the application, but the results presented here will act as a guide in the determination.

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