We propose to use a broad-bandwidth laser source for storing and retrieving multiple holograms in a photorefractive material. Each storage address is defined by a specific spectral encoding of the reference beam. The validity of the spectral encoding method is tested in a preliminary experiment. © 1996 Optical Society of America.

1. INTRODUCTION
Holographic materials are well known for their ability to record phase laws, and holograms can act as phase filters. Moreover, many holograms can be recorded in the same sample. These properties have been used for data storage. One problem is to retrieve selectively each piece of recorded information without any cross-talk effect.

Cross-talk effects can be avoided by use of various methods. The most widely used methods take advantage of the Bragg selectivity of thick holograms. Independent addresses can be specified by the angle of incidence of the reference beam or by its temporal frequency. In this case, spectrally selective materials can also be used.

A spatial phase-encoding method has also been proposed in which, for recording each datum, a set of reference beams with different angles of incidence simultaneously illuminates the storage material. Each beam is affected by a phase shift. Orthogonal phase sets guarantee the independence of the data addresses.

We propose a spectral phase-encoding method. The necessary broad-frequency spectrum is obtained by use of optical pulses of short coherence time (in the picosecond or the femtosecond range). A previous study examined this method in the case of spectrally selective materials. Here we consider the case of nonfrequency-selective materials (thick holograms in photorefractive crystals).

2. GENERAL PRINCIPLE OF DATA STORAGE AND ADDRESSING BY A SPECTRAL PHASE LAW
This principle has already been presented in Ref. 6. Let us briefly review it.

The object and the reference optical pulses have the same spectral amplitudes, denoted by $a(v)$. On the reference pulse the spectrum has been encoded, and its spectral amplitude becomes $a(v)C_i(v)$. By holography it is possible to realize the spectral filter

$$a(v)[a^*(v)C^*(v)].$$

Several object pulses can be recorded in the same holographic material, acting as a multiple phase filter for the different codes. In the readout step this hologram is illuminated by one of the reference beams, of spectral amplitude $a(v)C_j(v)$. The spectral amplitude of the reconstructed beam for the $i$th hologram is given by

$$a_r(i) = |a(v)|^2a(v)C_i^*(v)C_j(v),$$

corresponding to a temporal signal

$$S_{ri}(t) = \int a_r(i)\exp(j2\pi vt)dv.$$

In the considered picosecond or femtosecond range, the temporal signal cannot be directly studied. Instead, we record the field cross correlation of the signal with a time-delayed replica of the object field. As a function of the delay $\tau$, the cross-correlation function is expressed as

$$A(\tau) = \int |a(v)|^4C_i^*(v)C_j(v)\exp(j2\pi \tau v)dv.$$

The $\tau$ dependence of the cross-correlation function reflects the temporal evolution of $S_{ri}(t)$. Cross correlation works as an optical sampling procedure that is gated by the time-delayed replica of the object field. The time resolution of the technique is given by the coherence time of the light, which coincides with the inverse spectral width of the laser.

Let us assume that the spectral intensity has a constant value, limited by a spectral window of width $\Delta \mu$, over the useful part of the encoding function. At zero delay we obtain

$$A(0) = \int C_i^*(v)C_j(v)dv = \Delta \mu \delta(i - j) \quad (1)$$

if we assume the encoding functions to be orthogonal. The signal recovered at $\tau = 0$ is well selected: Each recorded hologram can be read only by its own reference.
beam. Note that such a method does not impose coherent Fourier-transform pulses because the result depends only on the spectrum intensity.

Practical Realization of the Spectral Phase Encoding

We use a method previously applied by Wiener for pulse shaping (Fig. 1). The laser beam is angularly dispersed by a diffraction grating G1 set in the object focal plane of a lens L1. Because of the finite size of the laser beam, each spectral frequency gives a diffraction pattern \( p[X - K(v - v_0)] \) in the image focal plane of L1, where \( X \) is the abscissa in this plane, \( v_0 \) is the central frequency of the spectrum, and \( K \) characterizes the spectral dispersion of the grating. In this plane a phase shaper based on a spatial light modulator is inserted to encode the pulse of the grating. In this plane a phase shaper based on a mask set in the spectrum angularly dispersed by the grating G1. The mask is a set of \( M \) jointed slits of identical width, which gives rise to a spatial distribution of 0 and \( \pi \) phase shifts (transmission factor of +1 and −1). The \( i \)th reference beam is then encoded by a mask

\[
\mathcal{M}_i(X) = f(X) \ast \sum_j \exp j \phi_{ij} \cdot \delta(X - p \Delta X),
\]

where \( f(X) \) represents a slit of width equal to \( \Delta X \), with \( \phi_{ij} \) being the 0 or \( \pi \) phase shift corresponding to the \( p \)th slit in the \( i \)th code. The resulting amplitude in the focal plane can be written as

\[
\mathcal{M}_i(X)p[X - K(v - v_0)].
\]

If the spectral dispersion is exactly compensated by a second grating, which stays in the image of the first one, for example, in a 4f setup (Fig. 1), the equivalent amplitude in the \( X \) plane becomes

\[
p(X) \mathcal{M}_i[X + K(v - v_0)].
\]

If the width of \( p(X) \) is small compared with that of \( f(X) \), the field amplitude can be written as

\[
p(X) \mathcal{M}_i[K(v - v_0)],
\]

and the encoding function is \( C_i(v - v_0) = \mathcal{M}_i[K(v - v_0)] \). It does not depend on the geometrical position of the hologram. In contrast, if the width of \( p(X) \) cannot be neglected, the encoding function depends on the position of the plane.

Just behind the second grating, or in any plane in the parallel beam beyond this grating, the encoding function is given by the convolution of the mask function by the diffraction spot on the mask.

In the image of the \( X \) plane, the encoding function is well defined by \( \mathcal{M}_i \) but varies with \( X \) as

\[
C_i(v - v_0, X) = p(X) \mathcal{M}_i[X + K(v - v_0)].
\]

The resolution of this spectral shaping technique as a function of the conjugation default of the mask is fully discussed in Ref. 12.

For such an array of \( M \) jointed elements, it has been shown that \( M \) orthogonal sets can be generated, satisfying the orthogonality condition as specified by relation (1). Thus the number of independent addresses with which data can be stored is given by \( M \).

Note that each address spans the entire spectral storage domain. The multiplexing character of this approach has to be distinguished from multichannel procedures in which each datum is stored at a specific position in the memory space. What is known as wavelength multiplexing is actually an intrinsically multichannel procedure even if it can be regarded as a spatial multiplexing technique.

In Sections 1 and 2 we have reviewed some general properties. In the following sections we consider the specific case of a photorefractive material.

3. REALIZATION OF A SPECTRAL PHASE FILTER IN A PHOTOREFRACTIVE MATERIAL

A. Recording of Uniform-Contrast Sinusoidal Fringes

Such fringes describe the hologram that is formed by plane waves in monochromatic light. The intensity spatial distribution can be written as

\[
I(z) = I_0[1 - m \cos(2\pi/d)z],
\]

where \( z \) is the coordinate on an axis perpendicular to the fringes; \( d \), the fringe spacing; and \( m \), the contrast of the fringes. The response of a photorefractive material is a spatial variation of the refractive index \( n \), whose amplitude is usually written as

\[
\Delta n = \Delta n_M[1 - \exp(-t/\tau)],
\]

where \( t \) is the exposure time and \( \Delta n_M \) and \( \tau \) are constant parameters determined by the material and the characteristics of the fringes.

\( \Delta n_M \), the saturation index, does not depend on \( I_0 \) and is equal to \( m \Delta N \), where \( \Delta N \) is a constant of the material.

\( \tau \) does not depend on \( m \) and is proportional to \( 1/I_0 \).

Note that such a law can be understood as a competition between the increase of \( \Delta n \) by the modulated part of...
the intensity and an erasing effect related to its constant part, resulting in a saturation value of the refractive-index variation. Note also that, despite saturation, the value of $\Delta n$ always remains proportional to $m$, regardless of its value.

B. Interference Fringes between Two Beams with Broad-Spectrum Pulses

Let us consider two counterpropagating beams (Fig. 2). One of them is the object pulse, and the other is the spectrally encoded reference pulse. The holographic material is located in (or near) an image plane of the mask. We assume that inside the crystal the diffraction effects can be neglected. Then the temporal signal on the two beams can be written as

$$E_0(t, X) = p(X)\exp[j2\pi\nu_0(t - z/v)] \int a(\mu) \times \exp[j2\pi\mu(t - z/v)]d\mu,$$

$$E_R(t, X) = p(X)\exp[j2\pi\nu_0(t + z/v)] \times \int a(\mu)C_\mu(\mu)\exp[j2\pi\mu(t + z/v)]d\mu,$$

respectively, where $\mu = v - \nu_0, v$ is the light velocity inside the material, and $p(X)$ denotes the transversal field variation and does not vary inside the crystal.

As the holographic material has a response time that is long compared with the pulse duration, it is sensitive to the time-integrated intensity of the interference signal. The equivalent interference fringes are given by

$$W(z, X) = p(X)^2\left[\int |a(\mu)|^2d\mu + \int |a(\mu)|^2C_\mu(\mu)^2d\mu \right. + \exp[-j2\pi\nu_0(2z/v)] \times \left[|a(\mu)|^2C_\mu^*(\mu) \times \exp[-j2\pi\mu(2z/v)]d\mu + c.c.\right].$$

The interference term in Eq. (3) can be characterized by a complex visibility function $V(z)$:

$$V(z) = \int |a(\mu)|^2C_\mu^*(\mu)\exp[-j2\pi\mu(2z/v)]d\mu = \int E_0(t - 2z/v)E_R^*(t)dt.$$ 

The constant part of the exposure is

$$W_0 = \int |a(\mu)|^2d\mu + \int |a(\mu)|^2C_\mu(\mu)^2d\mu = \int |E_0(t)|^2dt + \int |E_R(t)|^2dt,$$

and the contrast $m(z) = 2V(z)/W_0$.

Note that at any point inside the hologram the buildup occurs during the overlap time, i.e., the short-pulse duration, whereas erasing occurs during the entire duration of pulses. This means that if, for identical pulses, the contrast factor $m$ becomes equal to 1 near $z = 0$, this value can never be reached if one of the pulse is encoded. We shall show in Subsection 3.C that this effect is compensated by the larger extension of the visibility curve and does not affect the diffraction efficiency, which is the same for any encoded readout beam.

C. Reconstructed Signal in the Image Plane $z = 0$

Let us consider a readout beam (identical to one of the reference beams), encoded by $C_\mu(\mu)$. Note that calculating the reconstructed field for each frequency and coherently adding the corresponding fields is equivalent to directly calculating the temporal reconstructed signal. We used the first method.

If we neglect the depletion and the absorption of the beam, the spectral amplitude field at a position $(z, X)$ inside the crystal, for a specific readout beam $j$, is

$$a_j(\mu, X, z) = p(X)a(\mu)C_j(\mu, X)\exp[j2\pi(\nu_0 + \mu)z/v].$$

Assuming low efficiency, we can consider the engraved hologram as a collection of contiguous thin gratings of identical thickness $dz$, each modulated by the term $\exp[-j2\pi\nu_02z/v]$, with a diffraction efficiency proportional to $V(z)$. According to Kogelnik, the diffraction efficiency of an elementary reflective grating is given by

$$\tanh(\pi\Delta ndz/\lambda) = \pi\Delta ndz/\lambda,$$

where $\Delta n$ is proportional to $m(z)$ and $\lambda$ is the wavelength inside the crystal. In the case in which the saturation index modulation has been reached,

$$\Delta n(z) = 2\Delta NV(z)/W_0.$$ 

By adding the contributions of the elementary gratings, we find that in the plane $z = 0$, the frequency spectrum of the reconstructed field spatially limited by $|p(X)|^2p(X)$ is

$$a_r(\mu, X) = (2\pi/\lambda)(\Delta N/W_0)\int a_j(\mu, X, z)V(z) \times \exp[-j2\pi\nu_0(2z/v)] \times \exp[j2\pi(\nu_0 + \mu)2z/v]dz.$$

It becomes

$$a_r(\mu, X) = (2\pi/\lambda)(\Delta N/W_0)(\nu/2)|a(\mu)|^2 \times a(\mu)C_\mu^*(\mu, X)C_j(\mu, X)$$

if the length of the crystal can be considered as infinite, that is, much larger than the useful length of $V(z)$. 

---

Fig. 2. Hologram recording with two counterpropagating beams near a focus.
We have seen that the spectral code for the image plane depends on the position $X$ in this plane. $C(\mu, X)$ can be replaced by $\mathcal{M}(X + K\mu)$, and the frequency spectrum becomes

$$a_i(\mu, X) = (2\pi/\lambda)(\Delta N/W_0)(v/2)|a(\mu)|^2a(\mu) \times \mathcal{M}_i^*(X + K\mu)\mathcal{M}_i(X + K\mu).$$

The frequency spectrum of the field cross correlation is

$$|a(\mu, X)|^4 \mathcal{M}_i^*(X + K\mu)\mathcal{M}_j(X + K\mu).$$

Then the field cross correlation $A(0)$ at a zero delay, for a constant value of the spectral density, is

$$A(0) \sim \int p(X)^4 \left[ \int \mathcal{M}_i^*(X + K\mu)\mathcal{M}_j(X + K\mu) d\mu \right] dX.$$

By substituting $X + K\mu$ with $\mu'$, we can see that the integral on $\mu'$ does not depend on $X$, which confirms that the finite dimension of $p(X)$ does not impose any limitation on the spectral resolution in our experiment, provided that the image of the mask is positioned inside the recording material.

4. DISCUSSION

A. Diffraction Efficiency

Let us first consider the energy that is diffracted by an engraved hologram that is read by an encoded field. The diffracted signal $S_d$ is given by

$$S_d = \int |a_i(\mu)|^2 d\mu = \int |E(t)|^2 dt.$$

Below we assume that the light-intensity source is uniformly distributed over the spectral bandwidth $\Delta \mu$. By using expression (4) for $a_i(\mu)$, we obtain

$$S_d = (\pi 2\lambda/\Delta \mu)^2 \Delta N^2 W_0^2 \int |C_i(\mu)C_j(\mu)|^2 d\mu/\Delta \mu.$$

If $C_i, C_j$ are constant or pure phase laws, then the integral is equal to $\Delta \mu$, and the signal is the same for any $j$. The resulting efficiency is written as

$$\eta = (\pi \Delta N/\lambda)^2 (v/\Delta \mu)^2$$

$$= (\pi \Delta NL/\lambda)^2 (v \tau_c/2L)^2$$

$$= \eta_{\text{cr}}(v \tau_c/2L)^2,$$

where $\tau_c = 1/\Delta \mu$ is the coherence time of the pulse; $L$, the crystal length; and $\eta_{\text{cr}}$, the cw diffraction efficiency. One of the major differences between cw and pulsed recording lies in the hologram dimension. It is limited by the crystal length or by the pulse coherence length $v \tau_c$ for cw or pulse recording, respectively. For femtosecond pulses this length becomes much shorter (roughly 10 $\mu$m) than $L$ (several millimeters).

Note that, as the crystal length determines the spectral resolution (minimal value for one element of the mask), the ratio of the crystal length to the pulse coherence length is the maximum number $M$ of possible addresses. This recording scheme can be compared with the angular phase-encoding technique. In the latter approach the readout beam intensity is shared among $M$ reference beamlets, the field amplitude of which is $\sqrt{M}$ lower than that of the initial beam. $M$ holograms are coherently recorded; the available refractive-index modulation of each of them is $\Delta N/\sqrt{M}$. Each beamlet is diffracted on a single hologram. The resulting $M$ signals combine into a single field that represents the retrieved data. The resulting diffraction efficiency is

$$\eta = [(\pi L/\lambda)(\Delta N/\sqrt{M})(M/\sqrt{M})]^2 = (\pi \Delta NL/\lambda)^2.$$

Thus, for the same number $M$ of addresses, the signal diffraction efficiency in our method is approximately $M^2$ times lower than in the angular-phase-encoding method. This result can be ascribed to the mutual incoherence of hologram recordings at different wavelengths. Although the storage material is simultaneously illuminated by the object and the reference pulses, each spectral component of the object pulse interferes with a single spectral component of the reference. In contrast, in the case of angular phase encoding, the entire object field interferes with any reference beamlet.

Assuming that the saturation value was reached—$10^{-3}$ for LiNbO$_3$—we calculated a theoretical diffraction efficiency (for one recorded hologram) of a few $10^{-2}$ for a phase mask, $10^{-3}$ for the amplitude mask that we used, and a spectrum width of 5 nm.

B. Selectivity

We have seen that the orthogonality of the encoding function results in the perfect selectivity of addressing if the engraved spectral filter is exactly proportional to the mask function. The conditions to be fulfilled are

1. uniform spectral distribution of the laser intensity over the encoding mask,
2. positioning of the recording material inside the image plane of the mask, and
3. crystal length much larger than the spatial extension of the visibility curve (approximately the equivalent length of the encoded pulse).

The first condition can be satisfied at the expense of energy loss. In our experiment some nonuniformity remains, and its effect is shown in the experimental data (Section 5). The important point is the finite length of the crystal.

We have seen [Eq. (2)] that the encoding function can be written as

$$f(\mu)*\Sigma (\exp j\phi_{\mu})\delta(\mu - p\Delta \mu) = f(\mu)*C_i(\mu).$$

The visibility curve is obtained as a Fourier transform (FT) of this encoding function as

$$V(z) = [\text{FT}[f(\mu)]]\text{FT}[C_i(\mu)] = \sin(2\pi z/v)[\text{FT}[C_i(\mu)].$$

Because of the finite length of the crystal, which can be written as a rectangle function $R(z)$, the equivalent visibility curve is

$$[R(z)\sin(2\pi z/v)][\text{FT}[C_i(\mu)],$$

which means that, in the engraved spectral filter, $f(\mu)$ has to be replaced by a convolution with a sinc function of the crystal length. It results in an enlargement of each spec-
tral element, which is a cause of cross talk, as discussed in Section 5. One can avoid this cross talk by increasing the distance between the mask slits.

Note that the role of this sinc function is well known in the case of wavelength multiplexing, where \( f(\mu) \) is a \( \delta \) function and one avoids the cross talk by placing the successive wavelengths in the zeros of the sinc that is centered on one of them.

5. EXPERIMENTAL METHODS

To check this principle of storage and addressing, we performed a preliminary experiment in which we used an amplitude mask instead of a spatial light modulator, with the aim being to check the spectral resolution of the system. The setup can be described as follows.

The laser source was a dye laser pumped by a YAG laser delivering broadband chaotic pulses at 620 nm of 10-ns duration and 5-nm bandwidth at a 15-Hz repetition rate. The coherence time of the pulse is thus \( \tau_c = 250 \text{ fs} \), which corresponds to a coherence length \( l_c \) of 80 \( \mu \text{m} \). The laser beam is split into an object pulse, which is sent directly to the sample, and a spectrally shaped reference pulse. The spectral shaping device is identical to the one developed by Weiner et al.\(^{11}\) (see Fig. 1). The shaped spectrum presents a grooved structure with 0.05-nm-wide bright slices every 0.4 nm. The two beams counterpropagate (Fig. 2), and the delay is adjusted so that they interfere at zero delay at the center of the sample, a 6-mm-thick Fe:LiNbO\(_3\) crystal. A chaotic pulse can be regarded as a train of elementary uncorrelated components of duration \( \tau_c \). After shaping by the 0.05-nm-wide slits of the spectral mask, each 80-\( \mu \text{m}\)-long subpulse in the reference beam is expanded into a 7.7-mm-long structure. In the crystal, it shrinks into a 3-mm-long hologram because of the counterpropagating scheme and because of the refractive index of LiNbO\(_3\) (\( n = 2.5 \)). Each stored hologram spans the same spatial domain. However, owing to the multiplexing character of the storage, the memory capacity is given by \( 2nL/l_c = 400 \). During readout the object beam is blocked, and the spectrally shaped reference pulse is diffracted on the engraved hologram. The diffracted signal energy is collected by a photomultiplier (Subsection 3.C) and is written as

\[
S_0 \alpha \int |a(\mu)|^2 |C_i(\mu)C_j(\mu)|^2 d\mu.
\]

The same mask is used for readout. One varies the readout function \( C_j(\mu) \) by scanning the mask position along the \( X \) direction in the focal plane of \( L_1 \). Figure 3(a) represents the signal intensity as a function of the spectral distance of the readout mask from its recording position. The irregularities in the variations of the peak intensities most probably arise from the nonuniformity of the spectral intensity over the 5-nm bandwidth. The constant background is ascribed to scattering of the readout beam on the beam splitter through which the diffracted beam is detected.

Figure 3(b) shows the signal intensity for the central peaks as a function of the spectral shift. The triangle width is close to the expected value. The signal intensity at a distance of one slit width from the recording position represents the cross talk. It appears to amount to approximately 3% of the peak signal intensity. This can be explained by the finite length of the crystal. For an infinite-length crystal, the signal intensity is an exact triangular function. The effect of finite crystal length, as was already explained above, is to convolve the slit function \( f(\mu) \) by a sinc function, which for an equivalent crystal length of 6 mm \( \times 2.5 \text{ mm} \) is 0.025-nm wide. The net
effect is a widening of the slits. In other words, the slits’ edges are smoothed because of cutoff of the high-frequency components of the visibility function.

6. SUMMARY

We have experimentally verified a temporal spectrum-shaping technique for addressing data in a photorefractive material. This technique has been previously used in persistent-spectral-hole-burning materials working at low temperature. Formal comparison has been worked out between the explored spectral phase encoding and the angular phase encoding. The scattering efficiency appears to be less affected by the number of the storage addresses in the latter case.

ACKNOWLEDGMENT

I. Lorgeré is grateful to the Société des Amis des Sciences for its financial support.

REFERENCES