Preparation of inverted medium and processing in the inverted medium

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Abstract

The processing of weak optical signals in spatial-spectral holographic (SSH) materials coherently inverted with optical frequency chirped pulses were investigated. Simulations and experimental studies in Tm\(^{3+}\):YAG were conducted to characterize the parameters of the frequency chirped laser pulse used to invert the SSH material in order to obtain high photon echo efficiency for SSH lidar processing. Collinear and angled beam geometries and single shot and accumulated processes were investigated. Echo efficiencies as high as 450% were measured, significantly higher than the typical stimulated photon echo efficiency of 10%.

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1. Introduction

Spatial-spectral holographic (SSH) processing using inhomogeneously broadband absorbers has been proposed for a variety of applications, including optical memory and signal processing [1–3]. Wide inhomogeneous and narrow homogeneous line widths [4] enable SSH materials to process wideband signals with high spectral resolution (time-bandwidth products over 10\(^6\)). Range-Doppler processing of broadband (multi-GHz) radar and lidar signals in SSH materials have been demonstrated with fine delay resolution (sub-nanosecond) and fine Doppler resolution (∼100 Hz) [5]. Recent work has applied SSH processing to lidar and shown that SSH enables the implementation of noisy pulses to enhance the lidar range resolution and decrease requirements on laser stability [6].

The SSH processing used in broadband lidar is akin to the commonly known stimulated photon echo (SPE). In SPE and SSH processing, the delay between the lidar transmit and return signals is recorded as a spectral grating in the SSH medium. The spectral grating can be accumulated to enhance the signal-to-noise-ratio (SNR) for weak input pulses [7]. In SPE processing, delay information is obtained from the timing and temporal profile of generated SPE. The SPE approach thus requires high bandwidth detectors and electronics on the back end to faithfully capture the delay information electronically, resulting in poor SNR. Recent work in SSH radar and lidar processing has used a new approach, where a frequency chirped laser pulse probes the accumulated spectral grating. This converts the high bandwidth grating into a low bandwidth output signal that can be processed with high SNR by commercially available detectors and digitizers. The inherent high bandwidth, and thus fine range resolution, of the lidar processor is not compromised by the frequency scan method of readout.

Typical lidar return signals are weak and are often amplified before processing. Rare-earth-doped fiber amplifiers can be used to amplify the signal, but these introduce amplified spontaneous emission (ASE) noise and degrade SNR of the system [8]. To enhance the sensitivity of SSH lidar processing, optical inversion of SSH materials before processing is investigated in this paper. Besides the gain afforded during processing, the selectively inverted frequency bands are expected to have lower ASE noise than a broadband ASE and provide spectral filtering of out-of-band noise. Various approaches to SPE in gain medium

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with collinear beams have been previously observed and studied [9–13]. Accumulated photon echoes (APE) were first observed in amplified medium using Er-doped fibers [9] and later observed in inverted medium using SSH material [13]. In Ref. [9], 1.53 μm optical pulses were coupled into Er-doped fiber placed in liquid helium. The fiber was continuously pumped with counter propagating mode-locked laser at 810 nm. In Ref. [13], Er3+: YSO was first incoherently pumped with 10 ms long pulse to achieve population inversion before the SPE sequence was applied to generate a photon echo. The goal of these investigations [9,13] was to maximize the strength of the SPE, rather than the efficiency and sensitivity of SPE process, as is important in lidar applications. Maximizing the SPE efficiency was previously carried out with an approach in which the reference pulse (first pulse in SPE sequence) was made strong enough to partially invert the medium and acted both as amplifier pump and coherent reference [10]. While this produced SPE efficiencies over 100%, much greater than the typical SPE efficiency of less than 10%, this method is amenable to neither high bandwidth nor accumulation, which are both needed for high dynamic range lidar processing.

In this paper, potential for inverted SSH materials to amplify, accumulate, and process weak lidar return signals using inverted optically thick Tm3+: YAG is investigated. In Section 2, a method that utilizes a frequency chirped pulse to produce frequency selective inversion in the medium is explored. The population inversion and the amplification gain of a weak optical signal are analyzed through simulations and experiments. In Section 3, SSH processing in gain medium is investigated. Two brief pulses are used to create a spectral grating in the gain medium, which is read out with the SPE probe method. The SPE efficiency in the inverted medium is studied with simulation and experiment for both single shot and accumulated processes. Throughout this paper, unless specified differently, the efficiency is defined as the ratio between the SPE and pulse 2 in the programming pulse sequence.

2. Inversion and unsaturated gain

Coherent pumping with a frequency chirped laser pulse is an efficient method for inverting an inhomogeneously broadened optically thick medium through rapid adiabatic passage [14]. Recently, frequency selection through adiabatic passage using complex hyperbolic secant pulses (CHS) was also explored [15]. In Ref. [15] attention is focused on selection of ions at various frequency bins without effecting population in the neighboring frequency bins. The idea of using chirps to select ions at a certain frequency or invert the medium over a certain bandwidth is appealing because an optical chirp of moderate power can drive the population in a two-level absorber from its ground state to nearly full excitation. Square enveloped linear frequency chirped laser pulses are often used for simplicity [16], though pulses with structured envelopes and/or frequency modulation result in a smooth inversion spectra over selected bandwidths [14,17]. Similar schemes at radio frequencies (RF) are common in NMR systems for spin inversion [18,19]. In this paper, smoothed chirped pulses, which consist of a linear chirp with a square amplitude envelope sandwiched between rising and falling edges smoothed by hyperbolic phase and amplitude functions, are used for population inversion. A smoothed chirp provides uniform inversion over nearly the full bandwidth of the chirp [20].

The population inversion, r3, in an optically thin ideal medium pumped by an ideal linear frequency chirp is given by Landau–Zener formula [16,21], where the dimensionless pumping strength, the chirp rate, μ = E0/h, and the chirp rate, κ = Bc/Tc, represent the chirp bandwidth and the chirp duration, respectively. The dimensionless pumping strength, O, is determined by the chirp’s Rabi frequency, Ω0 = √κℏ, and the chirp rate, κ = Bc/Tc, where Bc and Tc represent the chirp bandwidth and the chirp duration, respectively. The dipole moment of the laser pulse and μ denotes the dipole moment of the electronics transition. In this formula, the ideal medium is assumed to have infinite coherence time, T2, and very long excited state lifetime, T1.

2.1. Simulations

The ideal chirp provides uniform excitation over the full spectrum. Under the realistic conditions, however, the material parameters, such as the coherence time, the lifetime, and the optical thickness, must be taken into account. There is no analytic expression for propagation of a chirped optical field under these conditions. Therefore, a Maxwell–Bloch simulator [22], which numerically integrates the coupled Maxwell and Bloch equations for studying the coherent interaction of applied electric field with the optically thick material, was used to determine the inversion under realistic conditions.

The maximum gain for an inverted two-level absorber is a function of the material’s optical thickness, x0L. When total population inversion is achieved, the unsaturated amplification gain should be, G = exp(+x0L). Therefore, relatively optically thick media (x0L > 1) are preferred, though very thick media may be difficult to invert. In our simulation and experiments, we used the 793 nm transition of 0.1% Tm3+: YAG as the SSH material. After consideration of available maximum power and the Rayleigh range of the beam propagating through the medium, along with the chirp rate required for significant (>80%) inversion, x0L = 3.8 was chosen, corresponding to G = 45. The relevant properties of Tm3+: YAG are the excited state lifetime, T1 = 800 μs, the coherence time, T2 = 16 μs at T = ~5K, and the relaxation bottleneck state lifetime, T3 = 10 ms [4].

Our simulation explored the effects of the chirp duration and the coherence time on the achievable population inversion. Fig. 1 shows the averaged population inversion as a function of Rabi frequency for chirp rates of 1 and 0.5 MHz/μs with infinite T2 and with T2 = 16 μs. The chirp bandwidth, Bc, is fixed at 10 MHz, while the chirp
duration, $T_c$, is varied from 10 to 20 $\mu$s. Within these excitation durations, the effects of the population decay can be ignored. The population inversion $\langle r_2 \rangle$ is averaged over the thickness of the medium, which is related to the small signal gain as, $G = \exp(20L\langle r_2 \rangle)$. The solid curve from Landau–Zener theory at chirp rate of 0.5 MHz/$\mu$s is plotted for comparison. Landau–Zener results deviate significantly from simulation results for thick medium with infinite (stars) and finite (squares) coherence times as $T_c$ is varied from 10 to 20 $\mu$s.

The curves peak at less than 100% inversion. For $T_c = 16$ $\mu$s (dotted line with stars) chirps are for infinite coherence time while 10 MHz/10 $\mu$s (dashed line with circles) and 10 MHz/20 $\mu$s (dashed line with squares) chirps are for finite coherence time, $T_c = 16$ $\mu$s.

In Fig. 1. Simulation results show average population inversion, $\langle r_2 \rangle$, in an optically thick medium, $20L = 3.8$, with 10 MHz chirps as a function of Rabi frequency. Solid line is Landau–Zener theoretical result for 0.5 MHz/$\mu$s chirp. Simulation results for 10 MHz/10 $\mu$s (dotted line with dots) and 10 MHz/20 $\mu$s (dotted line with stars) chirps are for infinite coherence time while 10 MHz/10 $\mu$s (dashed line with circles) and 10 MHz/20 $\mu$s (dashed line with squares) chirps are for finite coherence time, $T_c = 16$ $\mu$s.

a chirp with chirp duration less than the excited state decay time, $\sim 0.8$ ms. Thus, in order to get broadband, $\sim 10$ GHz, inversion in Tm$^{3+}$: YAG a chirp rate of $\sim 10$ GHz/0.8 ms or higher is needed. Similar inversion will be obtained for different chirped pulses, if the chirps’ dimensionless parameter, $\Phi$, are the same. Thus, a 10 GHz/0.8 ms chirp pulse requires 3.54 times higher Rabi frequency than a 1 MHz/1 $\mu$s chirp pulse for similar inversion. For the current experimental setup with a 140 $\mu$m laser spot size, 10 GHz/0.8 ms chirp would require 875 mW power for more than 80% inversion towards the end of the pulse.

In the simulations so far, plane waves are assumed. In experiments, however, the Rabi frequency across the beam profile follows a Gaussian spatial distribution. In Fig. 5, which compares the simulation with the experimental results, the spatial Gaussian profile of the laser beam is taken into account in the simulations [23]. In general, the Gaussian beam profile results in lower gain.

2.2. Experiment

Unsaturated gain in inverted Tm$^{3+}$: YAG under various conditions is measured. Main elements of the experimental setup for both the gain measurement and the SPE that follow are shown in Fig. 3(a). The laser source is a frequency stabilized Ti–Sapphire laser at 793 nm, resonant with the $^2$H$_4$$^2$H$_2$ optical transition in 0.1% Tm:YAG, stabilized to a few kHz line width by locking the laser to a regenerative spectral hole [24]. The cryogenically cooled (4 K) crystal had an absorption length, $20L$, of 3.8 at the peak of a 20 GHz-wide absorption profile. The two beam paths were focused (140 $\mu$m beam diameter) and overlapped in the crystal in an angled (0.026 radians) beam geometry. The two acousto-optic modulators (AOM1 and AOM2), driven by an arbitrary waveform generator (AWG), were used to create the various input laser pulses needed for the experiment. In all gain measurement experiments, collinear configuration (AOM1 only) is used.

The medium is first excited with an inversion chirp and then the inverted medium is probed with a weak pulse. In a high gain material, it is important that the amplified output should be sufficiently weak that it does not saturate the end
of the medium within the rise time of the AOM (~50 ns). For calibration the gain, the probe pulse must be measured at 100% transmission. A special probe pulse was crafted, which consists of three sub-pulses, as shown in Fig. 3(b). The two sub-pulses on the ends are weak with equal power of 5.0 μW (7.0 kHz Rabi frequency) while the middle sub-pulse is strong (300 mW, corresponding to 1.5 MHz Rabi frequency). The front part of the first sub-pulse samples the full gain, before it starts to saturate the gain. The strong middle sub-pulse ensures that the medium (on resonance) is fully saturated before the last sub-pulse passes through the medium with 100% transmission. Time duration of each of these sub-pulses is much longer than the material coherence time as shown in Fig. 4. A typical probe pulse after passing through the gain medium is plotted in Fig. 4 (as captured on a digital scope) and shows the amplified front, strong middle and end sub-pulse. The small signal gain is calculated as the ratio of the peak power of the first pulse to the power at the end of the last sub-pulse. After reaching a peak, the first sub-pulse saturates the medium, leading to an exponential decay in gain with decay time ~15 μs (see insert in Fig. 4), which is much smaller than the excited state lifetime of 800 μs, but much larger than the rise time of the AOM, yielding a good measure of unsaturated gain.

Using this method, gain is measured as a function of the inverting Rabi frequency for various chirp durations, \( T_c = 5, 10, 20, \text{ and } 40 \mu s \), for a fixed bandwidth, \( B_c = 10 \text{ MHz} \). In Fig. 5 experimental results for small signal gain versus the inversion pumping strength, \( \Phi = \Omega_0/\sqrt{\kappa} \), are plotted. The Rabi frequency is calibrated for the center intensity of the Gaussian beam as measured from an optical nutation signal [25]. The simulated gain curve, solid line with dots, for \( B_c = 10 \text{ MHz} \) and \( T_c = 10 \mu s \) is also plotted in Fig. 5. The experimental points for 1 MHz/μs (diamonds) are consistent with the simulation, with peak gain of about 20, which correspond to about 75–80% inversion. Laser power was not sufficient enough to reach the peak gain for 2.0 MHz/μs (stars). For chirp rates of 0.50 MHz/μs (triangle dots) and 0.25 MHz/μs (circular dots), the maximum gains were 18 and 13, respectively. The experiment confirms that, with finite coherence times, faster chirp rates are needed to achieve higher maximum gains, but at the expense of higher Rabi frequencies.

### 3. SPE experiments in an inverted medium

In SSH lidar processing, the first pulse sent into the material is a replica of the transmitted waveform and the second pulse represents the weak lidar return. The first pulse, and thus the return, are not limited to brief pulses, which is one of the major advantages of SSH lidar processing [6]. In our experiments, brief pulses were used for simplicity. In broadband SSH lidar processing, the preferred output is obtained by frequency scanning the
grating produced by the first two pulses (no SPE is generated). Since our investigations are at relatively low bandwidths (≪GHz), grating production can be studied effectively with the SPE probe method. Two forms of grating formation in an inverted SSH material are investigated: a single shot case (only one pulse pair) representing a single transmit and receive and an accumulation case (with multiple pulse pairs) representing a typical lidar system.

3.1. Single shot SPE experiments

The three pulses in the single shot study were introduced to the SSH material in two geometries. In the first geometry, the pulses were introduced collinearly and all pulses travel through path 1 as shown in Fig. 3(a). The medium was inverted first with the smoothed chirp and then programmed with a single sequence of two collinear pulses of pulse areas $A_1\sim 0.3\pi$ and $A_2\sim 0.03\pi$, which mimics a weak lidar return, as shown in Fig. 3(c). Here the pulse area is the peak Rabi frequency of the pulse times its duration. The resulting grating was probed after a delay $\tau_R$ (see Fig. 3(c)) by a third pulse of pulse area $A_1\sim 0.5\pi$. The delay between the first and second pulses was kept significantly less than $T_2$. In the collinear case, the maximum observed echo efficiency was 400%, which is the highest SPE efficiency reported to date.

In the angled beam geometry, the medium was inverted with the smoothed chirp traveling along beam path 1. Then, after waiting for 50 μs, ($\gg T_2$, but $\ll T_1$), the first pulse and the third “probe” pulse along path 2 and the second pulse along path 1 (see Fig. 3(a)) were applied to the inverted medium. The angled beam geometry may have practical advantages, since the SPE is isolated from the probe pulse, but it can suffer from a reduction in efficiency due to less beam overlap in the crystal, holographic instability, and low diffraction efficiency. The maximum echo efficiency obtained with the angled geometry was 350% with pulse areas $A_1 = 0.1\pi$, $A_2 = 0.01\pi$, and $A_3 = 0.4\pi$.

3.2. Accumulation SPE experiments

In practice, the lidar system would consist of a repetitive sequence of transmit pulses and weak return pulses, which allows for a grating to be accumulated in the inverted SSH medium. In accumulation, pulses 1 and 2 with delay $T_{21}$ (see Fig. 3(c)) are repeatedly applied and a spectral grating builds up in the inverted medium. Pulse 3 probes the grating, as in the single shot case, and produces an accumulated SPE.

Stabilization of the laser frequency is particularly important in accumulation, so that the grating phase does not fluctuate and the gratings from each shot add up. Angled beam geometry in the accumulation experiments was used to get better spatial discrimination of the SPE. The first two pulses were repeated every 43 μs after an initial inversion of the SSH material by a 10 MHz bandwidth, 10 μs long chirped pulse. The pulse area of the second pulse was kept weak, between $\frac{1}{5}$ and $\frac{1}{10}$ of the first pulse, so that the second pulse did not play a role in saturating the gain and thus the measured SPE efficiencies should be independent of the strength of the second pulse. From 1 to 13 repetitions of the pulse pair was allowed to accumulate before the resulting grating was probed (after 43.0 μs of delay) by a probe pulse of area $\sim 0.5\pi$.

The echo efficiency as a function of the number of pulse pairs is plotted in Fig. 6, along with the simulation results. The pulse areas of the first and the third pulse were fixed at 0.052π and 0.47π, respectively. Two data sets with different areas of the second pulse, 0.0026π and 0.0052π show that the echo efficiency is independent of the strength of pulse 2 in this regime. Both data sets and the simulation, show a rise in efficiency with the number of shots and then a decay after about 6–8 shots, which corresponds to a peak at about 500 μs (comparable to 800 μs, upper state life time of Tm$^{3+}$: YAG), indicating that saturation did not dominate the efficiency roll off. Under the experimental conditions explored, the highest observed echo efficiency was 450%. We also studied the effect of reversing the roles of pulses 1 and 2 in the accumulated SPE experiment, so pulse 1 is now the weak pulse, and found that it had little effect on the maximum echo efficiency (defined here as the ratio of the SPE to pulse 1).

The fact that these efficiencies were obtained in a relatively linear regime implies that cumulative processing, as needed for SSH lidar processing, can be performed without significant distortion. However, if the beam powers are significantly increased, saturation effects will diminish the effectiveness of cumulative range processing in the material.
4. Conclusion

Simulations and experimental studies were conducted to optimize the chirp parameters for obtaining high SPE efficiencies for SSH processing in inverted media. Collinear and angled beam geometries and single shot and accumulated gratings were investigated. Echo efficiencies as high as 450% were measured, significantly higher than the typical SPE efficiency of 10%. Results confirm that inversion of the SSH material can yield increased echo efficiency and may be useful in SSH lidar processing systems. It is still an open question, particularly in terms of achieved SNR, whether the method used here to enhance the echo efficiency is better than post amplification of the SPE.

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