Exam 3 Topics

• DC Circuits
  • Current & Ohm’s Law (Macro- and Microscopic)
  • Power
  • Kirchhoff’s Loop Rules
  • Charging/Discharging Capacitor (RC Circuits)

• Magnetic Fields
  • Force due to Magnetic Field (Lorentz Force)
  • Magnetic Dipoles
  • Generating Magnetic Fields
    • Biot-Savart Law & Ampere’s Law
General Exam Suggestions

• You should be able to complete every problem
  • If you are confused, ask
  • If it seems too hard, you aren’t thinking enough
  • Look for hints in other problems
  • If you are doing math, you’re doing too much
• Read directions completely (before & after)
• Write down what you know before starting
• Draw pictures, define (label) variables
  • Make sure that unknowns drop out of solution
• Don’t forget units!
Current & Ohm’s Law

\[ I = \frac{dQ}{dt} \]

\[ \vec{J} \equiv \frac{I}{A} \hat{I} \]

Ohm’s Laws

\[ \vec{E} = \rho \vec{J} = \left( \frac{1}{\sigma} \right) \vec{J} \]

\[ \Delta V = IR \]

\[ R = \frac{\rho l}{A} \]
Series vs. Parallel

Series
- Current same
- Voltages add

Parallel
- Currents add
- Voltages same

\[ R_s = R_1 + R_2 \]
\[ \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \]
\[ \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \]
\[ C_P = C_1 + C_2 \]
Current, Voltage & Power

**Battery**
\[ \Delta V = +\varepsilon \]

**Resistor**
\[ \Delta V = -IR \]

**Capacitor**
\[ \Delta V = -Q/C \]

\[ P_{\text{supplied}} = I \Delta V = I \varepsilon \]

\[ P_{\text{dissipated}} = I \Delta V = I^2 R = \frac{\Delta V^2}{R} \]

\[ P_{\text{absorbed}} = I \Delta V = \frac{dQ}{dt} \frac{Q}{C} \]

\[ = \frac{d}{dt} \frac{Q^2}{2C} = \frac{dU}{dt} \]
Kirchhoff’s Rules

\[ I_1 = I_2 + I_3 \]

\[ \Delta V = -\oint \vec{E} \cdot d\vec{s} = 0 \]

Closed Path
(Dis)Charging A Capacitor

\[ I = \pm \frac{dQ}{dt} \]

\[ Q = C\mathcal{E} \left(1 - e^{-t/RC}\right) \]

\[ \sum \Delta V_i = \mathcal{E} - \frac{Q}{C} - IR = 0 \]

\[ Q_{\text{final}} = C\mathcal{E} - Q - RC \frac{dQ}{dt} = 0 \]

\[ I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} \]
General Comment: RC

All Quantities Either:

\[ \tau = \frac{1}{RC} \]

\[ \text{Value}(t) = \text{Value}_{\text{Final}} \left( 1 - e^{-t/\tau} \right) \]

\[ \text{Value}(t) = \text{Value}_0 e^{-t/\tau} \]

\( \tau \) can be obtained from differential equation (prefactor on \( d/dt \)) e.g. \( \tau = RC \)
Right Hand Rules

1. Torque: Thumb = torque, Fingers show rotation
2. Feel: Thumb = I, Fingers = B, Palm = F
3. Create: Thumb = I Fingers (curl) = B
4. Moment: Fingers (curl) = I Thumb = Moment (=B inside loop)
Magnetic Force

\[ \mathbf{F}_B = q \mathbf{v} \times \mathbf{B} \]

\[ d\mathbf{F}_B = I d\mathbf{s} \times \mathbf{B} \]

\[ \mathbf{F}_B = I \left( \mathbf{L} \times \mathbf{B} \right) \]
The Biot-Savart Law

Current element of length $ds$ carrying current $I$ (or equivalently charge $q$ with velocity $v$) produces a magnetic field:

$$
\mathbf{B} = \frac{\mu_0 q \mathbf{v} \times \mathbf{r}}{4\pi r^2}
$$

$$
\text{d}\mathbf{B} = \frac{\mu_0 I \text{d}\mathbf{s} \times \mathbf{r}}{4\pi r^2}
$$
Biot-Savart: 2 Problem Types

Notice that $r$ is the same for every point on the loop. You don’t really need to integrate (except to find path length)
Ampere's Law: \[ \int \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} \]

Long Circular Symmetry

(Infinite) Current Sheet

Solenoid = 2 Current Sheets

Torus/Coax
Problem 1: Wire Loop

A current flowing in the circuit pictured produces a magnetic field at point P pointing out of the page with magnitude B.

a) What direction is the current flowing in the circuit?
b) What is the magnitude of the current flow?
Solution 1: Wire Loop

a) The current is flowing counter-clockwise, as shown above.

b) There are three segments of the wire: the semi-circle, the two horizontal leads, and the two vertical leads. The two vertical leads do not contribute to the B field (ds || r). The two horizontal leads make an infinite wire a distance D from the field point.
Solution 1: Wire Loop

For infinite wire use Ampere’s Law:
\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc} \Rightarrow B \cdot 2\pi D = \mu_0 I \]

\[ B = \frac{\mu_0 I}{2\pi D} \]

For the semi-circle use Biot-Savart:
\[ d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{r}}{r^2} \quad r = \frac{D}{2} \text{ and } d\mathbf{s} \perp \hat{r} \]

\[ B = \int d\mathbf{B} = \int \frac{\mu_0}{4\pi} \frac{I d\mathbf{s} \times \hat{r}}{r^2} \]

\[ = \frac{\mu_0 I}{4\pi} \frac{1}{r^2} \left( \pi r \right) = \frac{\mu_0 I}{4r} = \frac{\mu_0 I}{2D} \]
Solution 1: Wire Loop

Adding together the two parts:

\[ B = \frac{\mu_0 I}{2\pi D} + \frac{\mu_0 I}{2D} = \frac{\mu_0 I}{2D} \left( 1 + \frac{1}{\pi} \right) \]

They gave us B and want I to make that B:

\[ I = \frac{2DB}{\mu_0 \left( 1 + \frac{1}{\pi} \right)} \]
Problem 2: RC Circuit

Initially C is uncharged.

1. When the switch is first closed, what is the current $i_3$?
2. After a very long time, how much charge is stored on the capacitor?
3. Obtain a differential equation for the charge on the capacitor
   (Here only, let $R_1=R_2=R_3=R$)

Now the switch is opened

4. Immediately after opening the switch, what is $i_1$? $i_2$? $i_3$?
5. How long before $i_2$ falls to 1/e of this initial value?
Solution 2: RC Circuit

Initially C is uncharged → Looks like short

\[ R_{eq} = R_3 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \Rightarrow i_3 = \frac{\varepsilon}{R_{eq}} \]
Solution 2: RC Circuit

After a long time, C is full → \( i_2 = 0 \)

\[
\begin{align*}
    i_1 &= i_3 = \frac{\varepsilon}{R_1 + R_3} \\
    Q &= CV_C = C \left( i_1 R_1 \right) = C \varepsilon \frac{R_1}{R_1 + R_3}
\end{align*}
\]
Solution 2: RC Circuit

Kirchhoff’s Loop Rules

Left: \(-i_3R + \varepsilon - i_1R = 0\)

Right: \(-i_3R + \varepsilon - i_2R - \frac{q}{c} = 0\)

Current: \(i_3 = i_1 + i_2\)

Want to have \(i_2\) and \(q\) only \((L-2R)\):

\[
0 = -(i_1 + i_2)R + \varepsilon - i_1R + 2(i_1 + i_2)R - 2\varepsilon + 2i_2R + \frac{2q}{c}
\]

\[
= 3i_2R - \varepsilon + \frac{2q}{c}
\]

\[
i_2 = + \frac{dq}{dt}
\]

\[
\frac{dq}{dt} = \frac{\varepsilon}{3R} - \frac{2q}{3RC}
\]
Solution 2: RC Circuit

Now open the switch.

Capacitor now like a battery, with:

\[ V_C = \frac{Q}{C} = \varepsilon \frac{R_1}{R_1 + R_3} \]

\[ i_1 = -i_2 = \frac{V_C}{R_1 + R_2} = \varepsilon \frac{R_1}{R_1 + R_3} \frac{1}{R_1 + R_2} \]
Solution 2: RC Circuit

How long to fall to 1/e of initial current? The time constant!

This is an easy circuit since it just looks like a resistor and capacitor in series, so:

$$\tau = (R_1 + R_2)C$$

Notice that this is different than the charging time constant, because there was another resistor in the circuit during the charging.
Problem 3: Non-Uniform Slab

Consider the slab at left with non-uniform current density:

\[ \mathbf{J} = J_o \frac{|x|}{d} \hat{k} \]

Find B everywhere
Solution 3: Non-Uniform Slab

Direction: Up on right, down on left

Inside: (at 0<x<d): \( \int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc} \)

\[
\int \mathbf{B} \cdot d\mathbf{s} = B \ell + 0 + 0 + 0
\]

\[
\mu_0 I_{enc} = \mu_0 \int \int \mathbf{J} \cdot d\mathbf{A} = \mu_0 \int_0^x \frac{J_0 x}{d} \ell \, dx
\]

\[
= \mu_0 \frac{J_0 \ell x^2}{d} \frac{1}{2}
\]

\[
B = \mu_0 \frac{J_0 x^2}{d} \frac{1}{2} \quad \text{up}
\]
Solution 3: Non-Uniform Slab

Direction: Up on right, down on left

Outside: \((x > d)\):

\[
\int \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}
\]

\[
\int \mathbf{B} \cdot d\mathbf{s} = B \ell + 0 + 0 + 0
\]

\[
\mu_0 I_{enc} = \mu_0 \int \int \mathbf{J} \cdot d\mathbf{A} = \mu_0 \int_0^d \frac{J_0 x}{d} \ell dx
\]

\[
= \mu_0 \frac{J_0 \ell}{2} \frac{d^2}{d}
\]

\[
B = \frac{1}{2} \mu_0 J_0 d \text{ up}
\]
Problem 4: Solenoid

A current $I$ flows up a very long solenoid and then back down a wire lying along its axis, as pictured. The wires are negligibly small (i.e. their radius is 0) and are wrapped at $n$ turns per meter.

a) What is the force per unit length (magnitude and direction) on the straight wire due to the current in the solenoid?

b) A positive particle (mass $m$, charge $q$) is launched inside of the solenoid, at a distance $r = a$ to the right of the center. What velocity (direction and non-zero magnitude) must it have so that the field created by the wire along the axis never exerts a force on it?
Solution 4: Solenoid

SUPERPOSITION
You can just add the two fields from each part individually

a) Force on wire down axis
Since the current is anti-parallel to the field produced by the solenoid, there is no force (F=0) on this wire

b) Launching Charge q
The central wire produces a field that wraps in circles around it. To not feel a force due to this field, the particle must always move parallel to it – it must move in a circle of radius $a$ (since that is the radius it was launched from).
b) Launching Charge $q$

So first we should use Ampere’s law to calculate the field due to the solenoid:

$$\int \mathbf{B} \cdot d\mathbf{s} = Bl = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{l} = \mu_0 nI$$ up the solenoid

Now we just need to make a charge $q$ move in a circular orbit with $r = a$:

$$\mathbf{F}_B = q \mathbf{v} \times \mathbf{B} = qvB = m\frac{v^2}{r} = m\frac{v^2}{a}$$

$$v = \frac{qBa}{m} = \frac{q\mu_0 nIa}{m}$$ out of the page
Problem 5: Coaxial Cable

Consider a coaxial cable of with inner conductor of radius $a$ and outer conductor of inner radius $b$ and outer radius $c$. A current $I$ flows into the page on the inner conductor and out of the page on the outer conductor.

What is the magnetic field everywhere (magnitude and direction) as a function of distance $r$ from the center of the wire?
Solution 5: Coaxial Cable

Everywhere the magnetic field is clockwise. To figure out the magnitude use Ampere’s Law:

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}} \Rightarrow B \cdot 2\pi r = \mu_0 I_{\text{enc}} \]

\[ \Rightarrow B = \frac{\mu_0 I_{\text{enc}}}{2\pi r} \]

Drawn for \( a < r < b \)

The amount of current penetrating our Amperian loop depends on the radius \( r \):

\[ r \leq a: \quad I_{\text{enc}} = I \frac{r^2}{a^2} \quad \Rightarrow \quad B = \frac{\mu_0 I r}{2\pi a^2} \text{ clockwise} \]
**Solution 5: Coaxial Cable**

Remember: Everywhere

\[ B = \frac{\mu_0 I_{\text{enc}}}{2\pi r} \]  clockwise

\( a \leq r \leq b: \ I_{\text{Encl}} = I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r} \)  clockwise

\( b \leq r \leq c: \ I_{\text{Encl}} = I \left( 1 - \frac{r^2 - b^2}{c^2 - b^2} \right) \)

\[ \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r} \left( 1 - \frac{r^2 - b^2}{c^2 - b^2} \right) \text{ clockwise} \]
Solution 5: Coaxial Cable

Remember: Everywhere

\[ B = \frac{\mu_0 I_{enc}}{2\pi r} \text{ clockwise} \]

\[ r \geq c: \quad I_{Encl} = 0 \quad \Rightarrow \quad B = 0 \]