Photons: particle-like properties of radiation

Photoelectric effect

- A metal surface emits electrons when it absorbs light. This is called photoelectric effect.

- *Experimental detection.* Place two parallel metal electrodes inside a vacuum tube at distance $d$ from each other and apply voltage $V$ between them. Then connect an Ampere meter to the electrodes and measure current. In normal conditions no current is detected because the electrodes are separated by an insulator, empty space. However, if monochromatic light is shined on one of the electrodes (cathode), current may be detected depending on the voltage and light frequency $\nu$. This is possible because electrons can be ejected from the cathode by the absorption of light, and then ballistically accelerated by the electric field $E = V/d$ through the empty space between electrodes, until they hit the other electrode (anode).

- The ejection of an electron is subject to energy conservation: $T_{\text{out}} = E_0 - W$. Here, $T_{\text{out}}$ is the kinetic energy of the electron immediately after ejection, $E_0$ is the amount of absorbed radiation energy, and $W > 0$ is the amount of energy required to remove an electron from the electrode - electrons are bound to the metal bulk by the positively charged ions, and it takes a finite energy to extract them against Coulomb forces. Electrons at different locations and in different states of motion can have different $W$. The smallest possible $W$ (the minimum energy cost to eject an electron) is called work function.

- Once ejected, an electron is accelerated ballistically and acquires additional energy $eV$ by the time it reaches the other electrode (assuming $V > 0$).

- If $V < 0$, electrons are slowed down and lose kinetic energy, but those with $T_{\text{out}} > e|V|$ will be energetic enough to reach the anode. A large enough negative voltage $|V| > V_0 = e^{-1}T_{\text{out}}$ will stop the current (this is called stopping voltage).

Expectations from classical physics

- Electrons just beneath the cathode surface feel the oscillatory electric field of the incident radiation and accelerate in an oscillatory manner according to the Newton's law: $m_e \ddot{x}(t) = eE(t)$, where $E(t) = E_0 |\psi| \cos(2\pi \nu t)$ at the surface ($|\psi|$ is a unit-vector parallel to the surface). In this process electrons accumulate kinetic energy and at some point become energetic enough to overcome the work function barrier $W$ and escape from the cathode.

- Energy density carried by the electromagnetic field is proportional to $|E_0|^2$ according to Maxwell equations. Therefore, one can expect $E_0 \propto |E_0|^2$, so that the amount of absorbed energy from radiation is proportional to the radiation intensity. Consequently, the kinetic energy of escaped electrons $T_{\text{out}}$ will grow with radiation intensity.

- The measured current of electrons that reach the anode grows with the energy of ejected electrons $T_{\text{out}}$. Hence, more intense radiation should produce larger current.

- The larger the electromagnetic amplitude $E_0$, the faster electrons gain energy. An average time $\Delta t$ it takes an electron to accumulate enough energy for the escape depends on $E_0$ and decreases when $E_0$ increases. If radiation is turned on abruptly, the measured current should lag by $\Delta t$ which decreases when radiation intensity increases.

- The measured current, stopping voltage and time lag $\Delta t$ are not expected to depend on the radiation frequency $\nu$, but should depend on the radiation intensity.

Experimental findings

- Early experiments showed that electric discharge between electrodes becomes enhanced when ultraviolet light illuminates the cathode.
• Millikan’s experiment in 1914 demonstrated that photoelectric effect violates the classical physics expectations.

• The stopping voltage was found to be a linear function of frequency. \( V_0 = a(\nu - \nu_0) \). It did not depend on radiation intensity at all! This implied that \( T_{max} = eV_0 \) was independent on radiation intensity, contrary to classical physics expectations.

• No current was detectable when \( \nu < \nu_0 \), implying that electrons do not get enough energy to be ejected if radiation frequency is too small.

• No time lag \( \Delta t \) between turning on radiation and detecting current was ever observed.

**Einstein’s theory of photoelectric effect**

• Influenced by early experiments Einstein correctly predicted what Millikan later observed in experiments.

• Einstein postulated that electromagnetic radiation propagates in lumps of energy, rather than in the form of a continuous wave. These lumps were later named photons.

• Energy of a photon is proportional to its frequency, while the number flux of photons in a beam is proportional to its intensity.

• The Einstein’s hypothesis is consistent with Planck’s theory of blackbody radiation if photon’s energy is \( E_\gamma = h\nu \), where \( h \) is Planck’s constant.

• Using energy conservation, we find that \( T_{out} = h\nu - W \). Since \( T_{max} = eV_0 > T_{out} \), the theory predicts that the smallest voltage which stops the current is \( V_0 = h\nu/e - W/e \). The minimum frequency for photoelectric effect is \( \nu_0 = W/h \).

• These predictions match experiments, and the value of Planck’s constant determined from photoelectric effect was the same as that obtained independently from the blackbody radiation measurements. The absence of a time lag in experiments is understood by assuming that an electron absorbs a photon instantly, there is no gradual energy build-up.

**The Compton effect**

• Compton’s experiment: Monochromatic X-rays of wavelength \( \lambda \) are shined onto a graphite sample. The sample scatters X-rays in all possible directions. The intensity \( I_X(\theta, \lambda') \) of radiation scattered at the angle \( \theta \) from the incident beam direction is measured as a function of wavelength \( \lambda' \) (to determine the spectrum of scattered radiation).

• Classical electrodynamics expects that charged particles in graphite absorb and emit electromagnetic radiation without altering its frequency \( \nu = c/\lambda \). Therefore, the scattered X-ray intensity should be completely reduced to only one wavelength, \( \lambda' = \lambda \) according to classical physics.

• The measured \( I_X(\theta, \lambda') \) was found to have two pronounced peaks: one at the wavelength \( \lambda \) of the incident radiation, and one at \( \lambda' = \lambda + \lambda_C(1 - \cos \theta) \), where \( \lambda_C = 0.0243\text{Å} \) is a constant called Compton wavelength.

**Theory by Compton and Debye**

• The Compton scattering is successfully explained by assuming that X-rays are well described as a stream of energetic photons which behave as particles and collide with loosely bound electrons in the graphite sample.

• X-rays are very energetic; the photon energy \( E_\gamma = h\nu \) is very large and assumed much larger than the binding energy of an electron in a graphite atom. Much of this energy is transferred to the electron in a collision, so that the electron is ejected from the atom at a relativistic velocity.
• Photons are relativistic particles under all circumstances, since they always move at the speed of light, \( v = c \). Their energy \( E = m_0c^2/\sqrt{1 - v^2/c^2} = h\nu \) can be finite only if their rest mass is zero, \( m_0 = 0 \). Consequently, the photon momentum is \( p = h\nu/c = h/\lambda \) which follows from \( E^2 = c^2p^2 + (m_0c^2)^2 \) since \( m_0 = 0 \).

• A loosely bound electron in an atom has much smaller energy than an X-ray photon, so that we can neglect its initial velocity. An X-ray photon scatters from an electron at rest for all practical purposes. Let us assume that the electron (initially at rest) is kicked by the photon to momentum \( p_e \) at the angle \( \varphi \) with respect to the incident beam direction. The incident photon has wavelength \( \lambda \) and scatters at the angle \( \theta \) with a larger wavelength \( \lambda' \) (which means smaller energy).

• The conservation of momentum reads:

\[
\frac{h}{\lambda} = \frac{h}{\lambda'} \cos \theta + p_e \cos \varphi \quad \text{(along the incident direction)}
\]

\[
\frac{h}{\lambda'} \sin \theta = p_e \sin \varphi \quad \text{(perpendicular to the incident direction)}
\]

The conservation of energy reads:

\[
\frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + \sqrt{(m_e c^2)^2 + p_e^2 c^2}
\]

• Since we want to compare with the experiment, we are interested in \( \lambda' - \lambda \). We could obtain this from the energy conservation equation if we knew \( p_e \). Therefore, let us determine \( p_e \) from the momentum conservation equations. The easiest way to do this is to isolate \( p_e \) in both momentum conservation equations on the right-hand side, and add the obtained equation squares:

\[
(p_e \cos \varphi)^2 + (p_e \sin \varphi)^2 = p_e^2 = \left( \frac{h}{\lambda} - \frac{h}{\lambda'} \cos \theta \right)^2 + \left( \frac{h}{\lambda'} \sin \theta \right)^2
\]

After expanding the squares we find:

\[
p_e^2 = \left( \frac{h}{\lambda} \right)^2 + \left( \frac{h}{\lambda'} \right)^2 - \frac{2h^2}{\lambda \lambda'} \cos \theta
\]

Now, substitute this into the energy conservation equation. The easiest thing to do is immediately get rid of the square root on the right-hand side:

\[
(m_e c^2)^2 + p_e^2 c^2 = \left( \frac{hc}{\lambda} - \frac{hc}{\lambda'} + m_e c^2 \right)^2
\]

\[
(m_e c^2)^2 + \left( \frac{hc}{\lambda} \right)^2 - \frac{2h^2c^2}{\lambda \lambda'} \cos \theta = \left( \frac{hc}{\lambda} \right)^2 + \left( \frac{hc}{\lambda'} \right)^2 - \frac{2h^2c^2}{\lambda \lambda'} + (m_e c^2)^2 + 2m_e c^2 \left( \frac{hc}{\lambda} - \frac{hc}{\lambda'} \right)
\]

We cancel out the repeating terms and simplify:

\[
2\frac{h^2c^2}{\lambda \lambda'} (1 - \cos \theta) = 2m_e c^2 \left( \frac{hc}{\lambda} - \frac{hc}{\lambda'} \right) = 2m_e c^2 \frac{hc}{\lambda \lambda'} (\lambda' - \lambda)
\]

Finally:

\[
\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)
\]

• We see that this matches the experimental observation, with the Compton wavelength being \( \lambda_C = h/m_e c \).
Discussion

- The subsequent experiments were also able to detect the kicked electron and measure its momentum and energy. The measurements were in quantitative agreement with the above theory.

- This firmly established the existence of photons and their particle-like properties at high energies.

- Compton scattering should be contrasted with Thomson scattering in which the scattered radiation has the same wavelength as the incident radiation. Thomson scattering can be described using the laws of classical electrodynamics and generally occurs when the incident radiation is not very energetic in comparison to the electron binding energy.

- How can particle-like properties of photons be reconciled with its wave-like properties at lower energies? Consider two cases of low energy particle-like photons.

- Case 1): a photon does not have enough energy to knock-out an electron out of its atom. Then, the whole atom, rather than just the electron, must absorb the momentum and energy lost by the scattered photon. But, an atom has a large mass $m_a$ (much larger than the electron mass $m_e$). Consequently, the atomic Compton wavelength $h/m_a c$ is very small and undetectable. The scattered photon has essentially the same wavelength as the incident one, as if the scattering were approximately classical (governed by Maxwell equations which must also conserve momentum and energy).

- Case 2): a photon has a wavelength from the visible light range. This may be enough to knock-out a loosely bound outer-shell electron from an atom. However, $\lambda'/\lambda \sim \lambda_C$ is now very small in comparison with the photon wavelength (visible light wavelengths are 380-750nm, while X-ray wavelengths are 0.01-0.1nm (hard, Roentgen) and 0.1-10nm (soft)). This is hard to detect.