Electronic Density of States Near a Fluctuating Quantum Vortex and Vortex Dynamics in d-wave Superconductors

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Abstract

Following various indications that quantum fluctuations of vortices may play a fundamental role in the physics of cuprates, we describe the mutual influence of nodal fermionic quasiparticles and quantum vortices in clean two-dimensional d-wave superconductors. A pinned vortex can execute zero-point quantum oscillations and affect the electronic local density of states (LDOS) in its vicinity. As a function of energy, LDOS at the vortex center shows no zero-energy peak, but develops small satellite features driven by transitions between different vortex eigenmodes. These results qualitatively agree with scanning tunneling microscopy (STM) measurements of the sub-gap LDOS in cuprates. Furthermore, the zero-point vortex motion also naturally leads to the observed periodic modulations in the spatial dependence of the sub-gap LDOS. The same microscopic model describes the influence of quasiparticles on vortex dynamics. At zero temperature, the quasiparticles introduce a renormalization of the vortex effective mass (of the order of an electron mass), and a universal sub-Ohmic damping of the vortex motion. At finite temperatures, vortex experiences Bardeen-Stephen friction, with a universal co-efficient proportional to temperature squared. Such a potential can result from the quantum zero-point motion. Such a potential can result from wave superconductor at zero temperature. The vortex is assumed to be described by a universal co-efficient proportional to temperature squared. Similar results qualitatively agree with scanning tunneling microscopy measurements of the sub-gap LDOS in cuprates.

The Model

We consider a single vortex coupled to quasiparticles in a clean d-wave superconductor at zero temperature. The vortex is assumed to experience a harmonic trapping potential in which it can carry out its quantum zero-point motion. Such a potential can result from interactions between vortices in a vortex lattice, or from a pinning impurity. Assuming that the Magnus force is much smaller that the vortex harmonic frequency.

The following plots show how this correction to the LDOS depends on energy at the vortex center:

Calculation of the Electronic Local Density of States

We define the operators $b^+_l$, $b_l$ ($\Phi(\{x_j\})$) that lower/raise the quantum numbers $n_l$ of the pinned vortex. By resolving the unity in terms of the vortex eigenmodes $|n, \mathbf{x}\rangle$, in the harmonic trap, we can write the Hamiltonian as $H = H_0 + H_{\text{int}}$, where the unperturbed Hamiltonian includes effects of the vortex zero-point quantum fluctuations:

while the perturbation describes resonant scattering of quasiparticles from the fluctuating vortex:

In order to gain qualitative understanding of the physics, we introduce several approximations and simplifications:

- Vortex core size is negligibly small (a few lattice spacings);
- Vortex trap and quasiparticle dispersion are isotropic ($v_x = v_y = 1$);
- Doppler shift due to the vortex supercurrents is not considered (it is expected to only enhance the effects that we discuss).

Then, the quasiparticle wavefunctions of $H_{\text{v}}$ shaped by the vortex zero-point quantum fluctuations, are characterized by the quantum numbers:

- $q$: charge (sign of energy)
- $l$: angular momentum (integer)
- $k$: radial wave-vector (real positive)

and their spectrum is gapless $\varepsilon_q$ at the gap nodes. There are no localized states in the vortex cores of a pure d-wave superconductor!

LDOS is obtained from the quasiparticle Green's function $G_{\mathbf{k},\mathbf{k}',\mathbf{x},\mathbf{x}'}(\omega)$:

There is a qualitative agreement with experimental STM observations [plot taken from B.W.Joosgenboer, K.Kadowaki, B.Revaz, M.Li, Ch.Renner, O.Fischer; Phys.Rev.Lett. 87, 267001 (2001)].

Results

Quasiparticle LDOS $\rho(\varepsilon, e)$, influenced solely by the vortex zero-point quantum oscillations, is a monotonic increasing function of energy at all points in space, for any vortex mass. But, resonant scattering of quasiparticles from the fluctuating vortex leads to interesting effects already at the one-loop level:

The energy scans of the full LDOS at gradually increasing distances from the vortex center exhibit small sub-gap peaks that vanish beyond the region inside which the vortex executes its zero-point quantum oscillations:

Conclusions

The LDOS due to the resonant scattering of quasiparticles from the quantum oscillating vortex shows remarkable similarity to the experimental STM observations. Absence of a zero-energy LDOS peak is an indication of the small core size (similar analysis with a large core size produces a zero-energy peak [L.Bartosch, S.Sachdev]). A small sub-gap peak appears in the LDOS at the energy $h v_{\text{eff}}^2/2m^*$ as a result of the resonant scattering of quasiparticles from the quantum vortex – there are no bound core states! Some secondary weak features also appear in the LDOS, including a discontinuity at the energy $h v_{\text{eff}}$ (which must be blunted at finite temperatures). The future STM experiments might be sensitive enough to detect these features, and allow measurements of the vortex mass and vortex trapping forces. If the Magnus force is comparable to the trapping force, the LDOS remains qualitatively the same, but the exact energy scales are modified. Spatial modulations of LDOS with checkerboard pattern were also found in related earlier studies.

Vortex Dynamics

Vortices have a bare “hydrodynamic” mass (finite in charged superfluids) due to the superfluid stiffness, and generally experience friction and Magnus forces. Influence of the nodal quasiparticles on vortex dynamics has been previously studied only semiclassically, which may not be adequate in underdoped phases since the gapless nodal quasiparticles behave as a “quantum critical system” and lack an intrinsic length-scale that needs to be large for semiclassics. Within the present model we can study how the nodal quasiparticles renormalize the vortex action at the purely quantum level.

Instead of specifying the vortex Hamiltonian $H_\mathbf{v}$, we formulate a path-integral for quasiparticles coupled to a single vortex at position $r(\tau)$, and integrate out the fermionic quasiparticle fields. The resulting effective vortex action in frequency domain is given by:

with “longitudinal dynamics” given by:

All co-efficients are found exactly in absence of the Doppler shift $n_\mathbf{v} = \frac{1 + 1 - T}{1 + T} A - T \log (T)$ $\mathbf{m}_0 \approx m_\mathbf{v} \approx \frac{2}{\epsilon_{\text{v}}}$ $m_\mathbf{v}$.

Vortex mass that originates from the quasiparticles is of the order of an electron mass. At $T=0$, quasiparticles do not contribute to vortex friction, or transverse forces on the vortex. If the quasiparticle density of states at zero energy is finite $\rho(0)$ (for example, due to disorder) then friction and transverse forces emerge controlled by the energy scale $\rho(0)$ instead of temperature. Small vortex friction and mass at low temperatures have important implications for the flux-flow in the “normal state” of d-wave superconductors.

References: LO DOS near a quantum vortex

Predrag Nikolić, Subir Sachdev, Lorenz Bartosch; cond-mat/0606001

References: spatial modulations of LDOS

Predrag Nikolić, Subir Sachdev; cond-mat/0604165

Acknowledgments:

We thank Z.Tesanović, M.Fisher, E.Demler, B.I.Halperin and A.Kolezhuk for useful discussions. This research was supported by the National Science Foundation under grant DMR-0537077.