Strongly Correlated Physics With Ultra-Cold Atoms

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Overview

- Ultra-cold atoms: the new frontier in strongly correlated physics
- Interacting fermions in the unitarity limit
- BCS-BEC crossover in uniform systems
- BCS-BEC crossover in optical lattices
- Rotating fermion quantum liquids near unitarity
- “Unification” with condensed matter
- Conclusions
The new frontier in strongly correlated physics

- Many-body physics: collective phenomena of many particles, $N \gg 1$
  - Examples: Fermi liquid, Bose-Einstein condensate (BEC)

- Strongly correlated physics: inadequate single-particle description
  - Phenomena caused by interactions
  - Hidden identity of individual particles (emergent phenomena)
  - Examples: Mott insulators, superfluidity

- Condensed matter physics
  - The physics of many-body and strongly correlated systems
Recent breakthroughs in atomic physics

- 1995: Bose-Einstein condensate
- 1998: Feshbach resonance
- 1999: Degenerate Fermi gas
- 2000: Vortex lattice
- 2002: Optical lattice SF-Mott
- 2003: Fermion condensates
- 2004: Tonks-Girardeau gas
- 2004: BEC-BCS crossover
- 2006: Correlations (statistics, KT...)


Images from Greiner and Ketterle labs
Ultra-cold fermions

- **Fermions**
  - Lithium \(^{6}\text{Li}\), Potassium \(^{40}\text{K}\)
  - Two lowest hyperfine-split states: “spin” up and down

- **Trapping**
  - Magnetic (Zeeman effect), optical: \(N \sim 10^4 - 10^6\)

- **Cooling**
  - Laser cooling (Doppler shift): \(T \sim 100\text{mK}\) + Sisyphus: \(T \sim 10\text{mK}\)
  - Evaporative: \(T \sim n\text{K}\) ..... \(T_f \sim \mu\text{K}\)

- **Probing**
  - Absorption images (density), spectroscopy, interference
Strongly correlated fermions

- Feshbach resonance
  - Resonant two-body scattering
  - Scattering length exceeds inter-particle distance

- Optical lattice
  - Suppressed kinetic energy, interactions win
  - Superfluid-insulator transition

- Fast rotation
  - Quantum liquids, quantum Hall effect
  - Vortex lattices, KT transition

Images from Ketterle lab
Challenges ahead

Help us understand condensed matter materials
  - One dimensional physics
  - Correlated insulators (Mott, antiferromagnets)
  - Superconductivity in large magnetic fields (FFLO)
  - Cuprates, frustrated magnets, quantum computing...

Physics not accessible in condensed matter materials
  - BEC-BCS crossover
  - Pauli-Clogston limit & FFLO states
  - Exotic phases of matter (topological phases)
Interacting fermions in the unitarity limit

- Universality: irrelevant microscopic details
  - a many-body effect near a 2\textsuperscript{nd} order phase transition
  - “unitary” scattering

\[ \sigma \approx 4\pi a^2 \quad , \quad ka \ll 1 \]

\[ \sigma \approx 4\pi a^2 \left( 1 - \frac{\tan(\alpha_0 a)}{\alpha_0 a} \right) \quad , \quad ka \ll 1 \]

\[ \sigma \approx \frac{4\pi}{k^2 + \alpha_0^2 \cot(\alpha_0 a)} \approx \frac{4\pi}{k^2} \quad , \quad \alpha_0 = \sqrt{2mV_0} \]
Quantum field theory at unitarity

\[ S = \int d\tau d^d x \left[ \psi_i^{\dagger} \left( \frac{\partial}{\partial \tau} + \frac{(-i \nabla - A)^2}{2m} - \mu + V(\mathbf{x}) \right) \psi_i \right. \\
+ h \left( \psi_i^{\dagger} \psi_i^{\dagger} - \psi_i^{\dagger} \psi_i \right) + N \frac{m\nu}{4\pi} \Phi^{\dagger} \Phi + \Phi^{\dagger} \psi_i \psi_i^{\dagger} + \Phi \psi_i^{\dagger} \psi_i 
\]

Theoretical Approaches
- Mean-field approximation
- Perturbation theory
- Renormalization group

BCS-BEC crossover in uniform systems

- Attractive interactions & pairing correlations
  - Weak $\Rightarrow$ many-body “bound” state, BCS superconductor
  - Strong $\Rightarrow$ two-body bound state, BEC condensate of molecules

- Unitarity limit @ Feshbach resonance
  - The strongest pairing correlations and quantum entanglement
  - Novel state uniquely accessible in atomic physics

- Fundamental questions
  - The evolution of states between BCS and BEC limits
  - New quantum phases
Rice: polarized fermionic superfluids

G.B.Partridge, W.Li, R.I.Kamar, Y.A.Liao, R.G.Hulet
Science 311, 503 (2006)

G.B.Partridge, W.Li, Y.A.Liao, R.G.Hulet, M.Haque, H.T.C.Stoof
MIT: polarized fermionic superfluids


**T=0 phase diagram with population imbalance**

- 1\textsuperscript{st} order superfluid-metal transitions: \( h_c = 0.807\mu + O(1/N) \)
- 2\textsuperscript{nd} order superfluid-insulator (vacuum) transition
- Smooth BEC-BCS crossover
- Uniform magnetized BEC superfluid phase for \( \mu < 0 \)
- Normal metallic phases with one or two Fermi seas

Pairing fluctuations in $T=0$ normal states

- Pairing fluctuations increase pressure
- Longer-lived pairs $\rightarrow$ larger pressure
- No pressure increase in the fully polarized state
Superfluid critical temperature

- 2\textsuperscript{nd} order superfluid-normal phase transition at $T=T_c$

\[
\frac{\mu}{T_c} = 1.50448 + \frac{2.785}{N} + \mathcal{O}(1/N^2) \quad \text{N=1 \ MC}
\]

\[
\frac{\varepsilon_F}{T_c} = 2.01424 + \frac{5.317}{N} + \mathcal{O}(1/N^2) \quad \text{7.33124 \ 6.579}
\]

\[
\frac{P/N}{(2m)^{3/2}T_c^{5/2}} = 0.13188 + \frac{0.4046}{N} + \mathcal{O}(1/N^2) \quad \text{0.53648 \ 0.776}
\]

Monte-Carlo:
BCS-BEC crossover in lattice potentials

- 2\textsuperscript{nd} order superfluid-insulator phase transition at $T=0$, $\hbar=0$
- Band-Mott insulator crossover at unitarity (s-wave)

Critical lattice depth

- Saddle-point approximation
  - Diagonalize in continuum space near unitarity
  - Single-band Hubbard models: only deep in BCS or BEC limits...
  - Fix density - completely filled bands

\[ V_c = \frac{\hbar^2}{ma_L^2} F_n(a_L \nu) \]

At unitarity:

- Our result: \( V_c \sim 70 E_r \)
- MIT experiment: \( V_c \sim 6 E_r \)

Finite temperature effects?
Critical velocity and “band”-supersolids

- Uniform system
  - Superfluid-metal: pairing at $q=0$
  - Universal critical velocity

$$q/\pi \approx 0.58 \sqrt{2m(\mu - \mu_0)}$$
$$\mu_0 \approx 1.48T$$

- Lattice system
  - Superfluid - band-insulator: pairing instability at $q \neq 0$

P. Nikolić, A. Burkov, A. Paramekanti; (unpublished)
“Band”-supersolid
- The effect of pairing between different bands
- Lattice symmetry breaking in the insulator due to pairing fluctuations?

Unconventional Mott insulators
- Extended repulsive interactions (Coulomb, unitarity?)
- Fractional number of Cooper pairs per site
- Spontaneous lattice symmetry breaking

Transition to superfluid
- Conventional (Landau-Ginzburg) $\rightarrow$ supersolid
- Unconventional: deconfined critical point
Rotating fermion liquids near unitarity

- No time-reversal symmetry: quantum limit
  - Superfluid $\rightarrow$ vortex lattice
  - Fermi liquid $\rightarrow$ fermionic quantum Hall state
  - Correlated insulators $\rightarrow$ many possibilities

- Strongly correlated quantum insulators
  - Quantum Hall liquid of Cooper pairs
  - Density wave (Wigner crystal) of Cooper pairs

- Finite temperature, disorder
  - Phase transitions or crossovers between different normal states?
  - Critical fluctuations
Insulators and superfluids

- Normal state \(\rightarrow\) quantum Hall insulator
  - Localized particles (cyclotron orbitals)
  - Discrete Landau levels
  - Macroscopic degeneracy: two particles per flux quantum

**Superfluid**

\[
\Phi((r)) = \Delta_0 e^{-2m\omega y^2} \theta_3 \left( \left( \pi \sqrt{3} m\omega \right)^{\frac{1}{2}} (x + iy) \right) e^{i\pi/3}
\]
**Pairing instability**

- Quantum Hall $\rightarrow$ superfluid
- $2^{\text{nd}}$ order (saddle-point)


Superfluids & Vortex lattice FFLO states

- Competing forces
  - Pairing, orbital, Zeeman
- FFLO-metals and FFLO-insulators

P.Nikolić; (unpublished)
Experimental signatures

- Trapped gasses
  - Sharp shell boundaries
  - FFLO: $\rho_s \neq 0 \& p \neq 0$
  - FFLO-insulator: quantized $p$
  - FFLO-metal: variable $p$

- Features
  - Polarized outer shells
  - FFLO rings, abrupt appearance

\[ P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \]
Vortex liquid

- Genuine phase at $T=0$
  - Vortex lattice potential energy: $\Delta^4_0$
  - Melting kinetic energy gain: $\log^{-1}(\Delta_0)$
  - 1st order vortex lattice melting as $\Delta_0 \rightarrow 0$
  - Low energy spectrum inconsistent with fermionic quantum Hall states
  - Non-universal properties (by RG)

“Unification” with condensed matter

- **Unification**
  - Particle physics → “truth” is at high energies
  - Condensed matter → “truth” is at low energies

- **Universal collective many-body phenomena**
  - cold atoms exhibit similar phenomena as electronic systems
  - cold atoms are “ideal” and exceptionally tunable
  - “solvable” theories which can be compared with experiments
  - much to learn about probing

- **Some unresolved problems**
  - Hubbard model
  - Pseudogap in cuprates
  - Frustrated magnets & exotic phases
Pseudogap in cuprates

- “Fluctuating” superconductivity
  - Massless Dirac fermions
  - Lattice + Coulomb repulsion + pairing
  - $d$-wave $\rightarrow$ no vortex core states
  - Light, friction-free vortices
  - Quantum vortex dynamics

- Competing orders
  - Due to vortex quantum motion


Quantum motion of vortices


- Theory of vortices and quasiparticles
  
P.N., S.Sachdev; Physica C 460, 256 (2007)

Unconventional phases

• Search for unconventional phases of matter
  • Engineering custom quantum systems
  • Ideal lattices, very little relaxation...
  • Tunability
  • Measurement techniques?

• Unconventional physics
  • Valence-bond solids, spin liquids
  • Topological phases
  • Quantum computing

Conclusions

- **Unitarity**
  - Novel strongly correlated physics, universality
  - BCS-BEC crossover with population imbalance

- **Interacting fermions in lattice potentials**
  - Band-Mott insulator crossover at unitarity
  - Supersolids & novel Mott insulators

- **Fast-rotating interacting fermions**
  - Vortex lattices & FFLO states
  - Vortex liquids
New phases due to Zeeman effect

- Breakdown of superconductivity
- Pauli-Clogston limit: $\mu_\uparrow - \mu_\downarrow \sim \Delta$

**FFLO: non-uniform magnetized superfluid**

**Breached pair (Sarma) phase**

$Q = k_{F\uparrow} - k_{F\downarrow}$

$Q = 0$