Strongly Correlated Topological Insulators

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Topological Train Tracks

- $\mathbb{Z}_2$ topology in 1D:
  - Odd # of defects in a loop (measurable)
  - Defects can be created or removed in pairs
  - Invariant: the presence of a “charm” element
Strongly Correlated TIs

• Low-energy electrons live on the TI boundary...
  – How would they be affected by strong interactions?
  – Are there realistic ways to correlate them?

• Correlated TI ultra-thin films
  – Triplet orders & SU(2) vortices
  – Incompressible quantum liquids (fractional TIs)

• Correlations on a Kondo TI surface (SmB$_6$)
  – 2D “topological” heavy fermion system
Rashba S.O.C. on a TI's Boundary

• Particles + static SU(2) gauge field

\[ H = \frac{(\mathbf{p} - gA)^2}{2m} + \cdots \]

SU(2) generators (spin projection matrices)

\[ A_\mu = (\mathbf{A}, A_0) = A^\alpha_\mu S^\alpha \]

Yang-Mills flux matrix ("magnetic" for \( \mu=0 \))

\[ \Phi^\mu = \epsilon^{\mu\nu\lambda} (\partial_\nu A_\lambda - igA_\nu A_\lambda) \]

D. Hsieh, et.al, PRL 103, 146401 (2009)
Y. Zhang, et.al, Nature Phys. 6, 584 (2010)

\[ \mathbf{A} = -mv(\hat{\mathbf{z}} \times \mathbf{S}) \]

\[ \Phi^0 = g(mv)^2 S^z \]

\[ H = \frac{p^2}{2m} + v\tau^z \hat{\mathbf{z}}(\mathbf{S} \times \mathbf{p}) + \Delta \tau^x \]

Cyclotron: \[ \omega_\Phi = \Phi^z_0/m \gg 100 \text{ meV} \]

SU(2) flux: \[ n_\Phi = \Phi^z_0/\hbar^2 \gg 2 \cdot 10^{15} \text{ m}^{-2} \]
Correlated TIs in Heterostructures

- Artificially engineer correlations in a TI film
  - A 2D band-insulator with pairing interactions and tunable chemical potential ⇒ pair condensation QCP

- Cooper pair device
  - TI film (say Bi$_2$Se$_3$)
  - Pairing by proximity to a superconductor
  - QCP tuned by gate voltage, SC's $T_c$

- Exciton device
  - TI film in a capacitor
  - Biased capacitor creates an exciton gas
  - QCP tuned by capacitor bias
Triplet Instabilities

- Pairing between electrons of opposite SU(2) charge
  - Cooper pairs or excitons have spin $\Rightarrow$ couple to the Rashba $\mathcal{A}$

PN, T. Duric, Z. Tesanovic
PRL 110, 176804 (2013)

PN, Z. Tesanovic
PRB 87, 104514 (2013)

PN, Z. Tesanovic
PRB 87, 134511 (2013)

Kondo Topological Insulators
Pairing Channels

• The minimal TR-invariant topological band-insulator
  – 2 spin states \((\sigma = \pm 1)\) \(\times\) 2 surface states \((\tau = \pm 1)\)
  – short-range interactions

Decouple all interactions \(\Rightarrow\) 6 Hubbard-Stratonovitch fields:

Spin-singlet: \(\phi_\pm, \phi_0\)
Spin-triplet: \(\eta_m\)

<table>
<thead>
<tr>
<th>(\mathcal{T}_r) translations</th>
<th>(\psi_{\tau\sigma}(k))</th>
<th>(\phi_n(k))</th>
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<tr>
<td>(\mathcal{R}_\theta) rotations</td>
<td>(\psi_{\tau\sigma}(\mathcal{R}_\theta k))</td>
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<td>(\eta_m(\mathcal{R}_\theta k))</td>
</tr>
<tr>
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<tr>
<td>(\mathcal{I}_t) time reversal</td>
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<td>(-\phi^\dagger_n(-k))</td>
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</tr>
<tr>
<td>(\mathcal{C}) charge U(1)</td>
<td>(e^{i\theta} \psi_{\tau\sigma}(k))</td>
<td>(e^{i2\theta} \phi_n(k))</td>
<td>(e^{i2\theta} \eta_m(k))</td>
</tr>
<tr>
<td>(\mathcal{S}) spin U(1)</td>
<td>(e^{i\sigma\theta} \psi_{\tau\sigma}(k))</td>
<td>(\phi_n(k))</td>
<td>(e^{i2m\theta} \eta_m(k))</td>
</tr>
<tr>
<td>local spin SU(2)</td>
<td>(W_{\sigma\sigma'}\psi_{\tau\sigma'}(k))</td>
<td>(\phi_n(k))</td>
<td>(U_{mm'} \eta_{m'}(k))</td>
</tr>
</tbody>
</table>
Effective Theory

• The most generic Cooper-pair action
  – Integrate out all fermion fields near the Cooper Mott transition
  – Spin-triplets feel spin-orbit coupling

\[
S_{\text{eff}} = \int d\tilde{t} \, d^2r \left\{ (\partial_\mu \phi)^\dagger \hat{K}_s (\partial_\mu \phi) + \phi^\dagger \hat{t}_s \phi + K_t \left[ (\partial_\mu - iA_\mu) \eta \right]^\dagger \left[ (\partial_\mu - iA_\mu) \eta \right] + t_t \eta^\dagger \eta + U_t (\eta^\dagger \eta)^2 + U_{s,\sigma_1\sigma_2\sigma_3\sigma_4} \phi_{\sigma_1}^\dagger \phi_{\sigma_2}^\dagger \phi_{\sigma_3} \phi_{\sigma_4} + \left( \phi^\dagger \hat{t}_s \phi \right) (\eta^\dagger \eta) \right\} + \cdots
\]

Constructed from the SU(2) gauge symmetry (idealized)
There is SU(2) “magnetic” flux: \( A \propto \hat{z} \times \mathbf{L} \)

\[
\Phi^\mu = \epsilon^{\mu\nu\lambda} (\partial_\nu A_\lambda - iA_\nu A_\lambda) = (mv)^2 \delta_{\mu0} L^z
\]
Triplet Condensates & Vortices

– Rashba S.O.C.: momentum-dependent Zeeman
– A large-momentum mode has low energy

\[ H_0 = v \tau^2 \hat{z}(\mathbf{S} \times \mathbf{p}) + \Delta \tau^x \]

• Landau-Ginzburg picture
  – Helical “spin-current” T1 & T2 condensates
  – T1 phase can be TR-invariant
  – T1 breaks rotation and translation symmetries
  – T1 has metastable vortex clusters & lattices

PN, PRA 90, 023623 (2014)
Type-I Condensates

- Spin current without charge current

\[ \eta = \begin{pmatrix} \eta_\uparrow \\ \eta_0 \\ \eta_\downarrow \end{pmatrix} = \zeta \begin{pmatrix} e^{-i\theta} \cos \alpha \\ i\sqrt{2} \sin \alpha \\ e^{i\theta} \cos \alpha \end{pmatrix} \]

- Spin current densities & the Hamiltonian

\[ J_x = \frac{2\zeta^2}{m} \left[ \cos \theta \nabla \alpha + \frac{1}{2} \sin \theta \sin(2\alpha) \nabla \theta \right] \]
\[ J_y = \frac{2\zeta^2}{m} \left[ \sin \theta \nabla \alpha - \frac{1}{2} \cos \theta \sin(2\alpha) \nabla \theta \right] \]
\[ J_z = -\frac{2\zeta^2}{m} \cos^2 \alpha \nabla \theta \]

\[ H \to K \int d^2r \left[ (\nabla \alpha + \kappa \hat{z} \times \hat{\theta})^2 + \frac{1}{2} (\nabla \theta)^2 \right] \]

Rashba S.O.C. \Rightarrow \nabla \alpha \perp \hat{\theta}
Type-I Vortices

• Conservation laws: \([D_\mu, I_\mu] = 0\)

\[\nabla^2 \theta = -2mv \hat{\theta} \nabla \alpha \rightarrow 0\]
- no sources for \(J^z\), and \(J^{x,y} \perp \hat{\theta}\)

\[\nabla^2 \alpha = mv \hat{\theta} \nabla \theta\]
- \(J^z\) vortex is \(J^{x,y}\) source/drain

\[(\nabla \theta)(\nabla \alpha) = 0\]

• Neutrality: vortex quadruplets
  - vortices carry two “charges”
  - \(U(1) \theta\) (anti)vortex is bound to \(\nabla \alpha\) vector (anti)vortex

\[\nabla \theta \sim J^z\]
\[\nabla \alpha \sim J^{x,y}\]
\[\hat{\theta}\]
Type-I Vortex Structures

• Non-neutral clusters

• Domain wall

• Vortex lattice
  – unit cell is a quadruplet
  – square geometry
  – $\alpha$ changes by $n\pi$ between singularities
    $\Rightarrow$ rigid (meta)stable structure
    $\Rightarrow$ one ($n = 1$) vortex per SU(2) “flux quantum”
Stability of Vortex States

• Continuum: vortex cores are costly ⇒ uniform states

• Do vortex lattices ever win?
  – good candidates: metastable type-I structures
  – tight-binding lattice systems: vortex cores are cheap (if small)
  – entropy favors vortices (order by disorder, or vortex liquids)

• Microscopic lattice model
  – triplet pairing of $S = \frac{1}{2}$ fermions with Rashba S.O.C. on a square lattice bilayer (triplet superconductivity in a TI film)

$$H_0 = \sum_i \left[ -t \sum_{j \in i} \psi_i^\dagger e^{-i\tau^z A_{ij}} \psi_j + \psi_i^\dagger (\Delta \tau^x - \mu) \psi_i \right]$$

$$A_{ij} = -A_{ji}, \quad A_{i,i+\hat{x}} = \frac{mva}{2}\sigma^y, \quad A_{i,i+\hat{y}} = -\frac{mva}{2}\sigma^x$$
The Hamiltonian of TI Surfaces

\[ H = - \sum_{rr'} t_{r, r'} \hat{c}_r^\dagger e^{i \hat{r} \cdot \hat{r}'} \hat{A}_{r, r'} \hat{c}_{r'} \]

2D tight-binding model

\[ A_{r, r + \hat{x}} = a \sigma^y, \quad A_{r, r + \hat{y}} = -a \sigma^x \]

SU(2) gauge field on lattice links

- Singularity ("defect")
  Dirac point in \( E(k) \) at some energy

- Dirac points can be gapped only in pairs (TR symmetry)

- SmB\(_6\) (100 surface):
  M is gapped (bulk TI)

\[ H = \frac{(p - gA)^2}{2m} + \ldots \]

\[ A = -mv(\hat{z} \times S) \]

Continuum limit
Rashba spin-orbit coupling
\[ pA \propto \hat{z}(S \times p) \]

\[ \Phi^\mu = \epsilon^{\mu \nu \lambda} (\partial_\nu A_\lambda - ig A_\nu A_\lambda) \]

Yang-Mills (magnetic) flux

- SU(2) charge:
  \( g = \tau^z = \pm 1 \) helicity of spin-momentum locking

Kondo Topological Insulators
Correlated Lattice TI Films

• Inter-surface triplet pairing
  – Enhanced at large momenta by S.O.C.

$$\tau^z = +1$$ \hspace{1cm} \begin{array}{c}
\phi_0 \\
\eta_0 \\
\eta_+ \\
\eta_-
\end{array}$$

$$\tau^z = -1$$ \hspace{1cm} \begin{array}{c}
\phi_+ \\
\eta_0 \\
\eta_+ \\
\eta_-
\end{array}$$

• Finite-momentum condensate:
  – SU(2) vortex lattices

$$H = \frac{(p - gA)^2}{2m} + \cdots$$

$$\mathcal{A} = -mv(\hat{z} \times \mathbf{S})$$

$$\Phi^\mu = \epsilon^{\mu\nu\lambda}(\partial_\nu A_\lambda - igA_\nu A_\lambda)$$

Similar to:
W.S.Cole, S.Zhang, A.Paramekanti, N. Trivedi,
PRL 109, 085302 (2012)
Competing Orders

$|Q|$
Vortex Lattice Melting

• Quantum fluctuations
  – Positional fluctuations of SU(2) vortices
  – Grow when the condensate is weakened (by tuning the gate voltage)
  – Eventually, 1\textsuperscript{st} order phase transition (preempts the 2\textsuperscript{nd} order one)

• Vortex liquid
  – Particles per flux quantum \( \sim 1 \)
  – Fractional TI

\[ \text{Cyclotron: } \omega_\Phi = \Phi_0^z / m \gg 100 \text{ meV} \]
\[ \text{SU(2) flux: } n_\Phi = \Phi_0^z / h^2 \gg 2 \cdot 10^{15} \text{ m}^{-2} \]

• Fractionalization by vortex lattice melting?
  – Numerical evidence: N. Cooper, etc. … U(1) bosonic quantum Hall
  – Field theory indications: a generalization of Chern-Simons
Effective Theory of Fractional TIs

• Landau-Ginzburg theory (dual Lagrangian of vortices)

\[
\mathcal{L}_{\text{LG}} = \frac{K}{2} \left| (\partial_\mu - iB_\mu)\psi \right|^2 - t|\psi|^2 - t'\psi^\dagger \Phi_0 \psi + u|\psi|^4 + v|\psi^\dagger \gamma^a \psi|^2 + v'|\psi^\dagger \Phi_0 \psi|^2 \\
+ \frac{1}{8\pi^2} \text{tr} \left[ Q^{-2} (\Phi_B - \Theta_\mu)^2 \right]
\]

Maxwell term (density fluctuations)

• Topological term

– equation of motion ⇒ “drift currents” in U(1)xSU(2) E&M field

\[
\mathcal{L}_t = \frac{i}{8} \psi^\dagger \epsilon^{\mu\nu\lambda} \left[ D_\mu \left\{ D_\nu, \Phi_0 \right\} D_\lambda + \left\{ D_\mu D_\nu D_\lambda, \Phi_0 \right\} \right] \psi
\]

Levi-Civita tensor anti-commutators flux matrix

\[
D_\mu = \partial_\mu - iA_\mu \quad \Phi^\mu = \epsilon^{\mu\nu\lambda} (\partial_\nu A_\lambda - iA_\nu A_\lambda)
\]

PN, PRB 87, 245120 (2013)

Duality

• Boson-vortex duality in (2+1)D

Exact if:
– particles are bosons
– $S^z$ is conserved

Conjecture:
– generalize by symmetry
– fermionic particles?

• Particle Lagrangian

$$\mathcal{L}_{p\text{ LG}} = \xi^\dagger (\partial_0 - iA_0)\xi + \left[(\partial_i - iA_i)\xi\right]^\dagger \tilde{K} \left[(\partial_i - iA_i)\xi\right] - \xi^\dagger \tilde{t} \xi + \tilde{u}|\xi|^4 + \tilde{v}|\xi^\dagger \gamma^a \xi|^2$$

$$\mathcal{L}_{p\text{ t}} = \frac{i}{8} \xi^\dagger \varepsilon^{\mu\nu\lambda} \left[ \partial_\mu \left\{ \partial_\nu, \Theta_0^{-1} \right\} \partial_\lambda + \left\{ \partial_\mu \partial_\nu \partial_\lambda, \Theta_0^{-1} \right\} \right] \xi$$

Kondo Topological Insulators
Abelian Quantum Hall Liquids

• Low-energy dynamics:

\[
\xi(\mathbf{r}) = \begin{pmatrix} C_{\uparrow} e^{i\varphi_{\uparrow}(\mathbf{r})} \\ C_{\downarrow} e^{i\varphi_{\downarrow}(\mathbf{r})} \end{pmatrix}, \quad \psi(\mathbf{r}) = \begin{pmatrix} \sqrt{\rho_{\uparrow}} e^{i\theta_{\uparrow}(\mathbf{r})} \\ \sqrt{\rho_{\downarrow}} e^{i\theta_{\downarrow}(\mathbf{r})} \end{pmatrix}
\]

\[
\mathcal{L}_{\text{CS}} = \sum_{s} \left[ \frac{i\sigma_{s}}{4\pi m} \epsilon^{\mu\nu\lambda} b_{s\mu} \partial_{\nu} b_{s\lambda} - \frac{i}{4\pi m} \epsilon^{\mu\nu\lambda} b_{s\mu} \partial_{\nu} A_{\lambda}^z + \frac{1}{8\pi^2 q^2} (\epsilon^{\mu\nu\lambda} \partial_{\nu} b_{s\lambda} - \Theta_{s0} \delta_{\mu0})^2 \right]
\]

Chern-Simons \quad Maxwell

\[
b_{s\mu} = \partial_{\mu} \theta_{s} \quad \rho_{s} = \text{const.} \quad \Theta_{0} = \Phi_{0}
\]

• Fractionalization

– Measured charge, spin ⇒ quantized \( j_0 \, dA = 1 \)

– Dual vorticity ⇒ quantized \( \oint_{dC} d\mu \tilde{j}_\mu = 2\pi m \)

– Combine in quantum spin-Hall liquids ⇒ Laughlin sequence, quantized \( \rho_{s} = \langle \psi_{s}^{\dagger} \psi_{s} \rangle \)
Hierarchical States

• Emergent symmetries in the low-energy dynamics
  – Multiple vortex “flavors”

\[
\psi = (\psi^1, \ldots, \psi^n)
\]

\[
j^i_\mu = \sum_j Y^{ij} \epsilon^{\mu\nu\lambda} \partial_\nu \tilde{J}_\lambda^j
\]

[\Phi_0, \Theta_0] \neq 0

Laughlin states:

\[
Y_{ij} \propto \delta_{ij} \quad \Theta_0 = \Phi_0
\]

\[
\frac{i}{8} \psi^\dagger \Phi_0 \epsilon^{\mu\nu\lambda} D_\mu D_\nu D_\lambda \psi + \cdots
\]

\[
\frac{i}{8} \xi^\dagger \Theta_0^{-1} \epsilon^{\mu\nu\lambda} \partial_\mu \partial_\nu \partial_\lambda \xi + \cdots
\]

• SU(2) hierarchy for spin-\(S\) particles

\[
\psi = \left( (\psi^1_{-S}, \ldots, \psi^1_S), \ldots, (\psi^n_{-S}, \ldots, \psi^n_S) \right)
\]

\[
j^i_\mu = \epsilon^{\mu\nu\lambda} \partial_\nu \left( Y^{ij}_{00} \tilde{J}_\lambda^j + Y^{ij}_{0a} \tilde{J}_\lambda^a \right)
\]

\[
j^{ia}_\mu = \epsilon^{\mu\nu\lambda} \partial_\nu \left( Y^{ij}_{a0} \tilde{J}_\lambda^j + Y^{ij}_{ab} \tilde{J}_\lambda^b \right)
\]
Rashba Spin-Orbit Coupling

• Low-energy dynamics:
  – Helical modes: $S \perp p$

• Incompressible states:
  – Local density constraint: one free local parameter $\varphi(r)$ per mode

$$\xi^\dagger(r)\xi(r) = \text{const.} \quad \Rightarrow \quad \xi(r) = \hat{U}[r; \varphi(r)] \begin{pmatrix} C^+ \\ C^- \end{pmatrix}, \quad U^\dagger U = 1$$

$$\hat{U}_{\text{spin-Hall}} = e^{i(\varphi_0 + \varphi_z \sigma^z)}$$

$$\hat{U}_{\text{Rashba}} = e^{i(\varphi_0 + \varphi_x \sigma^x + \varphi_y \sigma^y)}$$

$$\mathcal{Z}_\mu = -i(\partial_\mu \hat{U})\hat{U}^\dagger$$

$$\mathcal{L}_P t = \frac{i}{8} \xi^\dagger \epsilon^{\mu\nu\lambda} \left[ \partial_\mu \left\{ \partial_\nu, \Theta_0^{-1} \right\} \partial_\lambda + \left\{ \partial_\mu, \partial_\nu \partial_\lambda, \Theta_0^{-1} \right\} \right] \xi \to -\frac{i}{8} \text{tr} \left\{ \mathcal{Z}_\mu, \Theta_0^{-1} \right\} \left\{ \Phi^\mu_Z, \xi \xi^\dagger \right\}$$
Kondo Insulators

• Materials:
  – SmB$_6$, YbB$_{12}$, Ce$_3$Bi$_4$Pt$_3$, Ce$_3$Pt$_3$Sb$_3$, CeNiSn, CeRhSb...

• Electron spectrum
  – Hybridized broad $d$ and narrow $f$ orbitals of the rare earth atom (e.g. Sm)
  – Heavy fermion insulators: $E_f$ is inside the hybridization bandgap
  – Coulomb interactions $\gg f$-bandwidth

• Correlations
  – $T$-dependent gap
  – Collective modes
Kondo TIs (SmB$_6$)

- Is SmB$_6$ a topological insulator?
  - Theoretical proposal: Dzero, Sun, Galtski, Coleman
  - Residual $T \to 0$ resistivity & surface conduction
  - Quantum Hall effect
  - Latest neutron scattering
  - Spin-sensitive ARPES


D.J. Kim, J. Xia, Z. Fisk arXiv:1307.0448

Collective Modes in SmB$_6$

- Sharp peak in inelastic neutron scattering
  - Nearly flat dispersion at 14 meV
  - Protected from decay (inside the 19 meV bandgap)
  - Theory: perturbative slave boson model

Neutron scattering experiment: Broholm group at IQM/JHU
Theory: Fuhrman, PN
Collective Mode Theory

• Perturbative slave boson theory
  – Applied to the Anderson model adapted by Dzero, et al.
  – Input: renormalized band-structure + 3 phenomenological param.
  – Output: mode dispersion & spectral weight in the 1st B.Z.

• Slave bosons
  – no double-occupancy of lattice sites by $f$ electrons

\[ f_\alpha = b^\dagger \psi_\alpha \]
\[ \sum_\alpha \psi_\alpha^\dagger \psi_\alpha + b^\dagger b = 1 \]
SmB₆ Theory + Experiment

- Match the theoretical & experimental mode spectra
  - Fit the band-structure and phenomenological param.
  - deduce the renormalized fermion spectrum

implied band inversion at X ⇒ a TI
Kondo TI Boundary

- A 2D Dirac heavy fermion system

SmB$_6$ metallic surface electron dispersion

In this cartoon:
- Dirac surface states are made from the near-$E_f$ portions of the bent bulk bands.

PN, PRB 90, 235107 (2014)

“heavy” surface electrons?

“light” surface electrons?

Hybridized regime

Local moment regime
Hybridized Surface Regime

• 2D perturbative slave boson theory
  – Slave bosons mediate forces among electrons
  – **attractive in exciton, repulsive in Cooper ch.**

  ![Basic process](image)

  Basic process:
  $f$-$d$ Hybridization assisted by a slave boson evolves into Kondo singlet upon localization

  \[
  \tilde{\Gamma}(q) = \cdots = \begin{array}{c}
  \text{slave boson renormalization}
  \end{array}
  \]

  \[
  \tilde{\Gamma}'(q) = \cdots = \begin{array}{c}
  \text{renormalization}
  \end{array}
  \]

• **Instabilities by nesting**
  – Magnetic (SDW) $\Rightarrow$ Dirac points perish
  – Superconductivity ($s^\pm$ or $d$-wave) $\Rightarrow$ Dirac points live

Recall: we have a collective mode right at these wavevectors
Surface Instabilities

• Weak-coupling orders
  – Cooper: $\Delta_k(q) = \langle f_k f_{-k} \rangle$
  – Exciton: $\Phi_k(q) = \langle f_k^{\dagger} f_{q+k} \rangle$

• Stronger coupling at $q = (\pi, \pi)$
  – The same triplets as in ultra-thin TI films

TR-invariant:

$$\Delta(\pi, 0) + i \Delta(0, \pi)$$

$$\left[ \Phi(\pi, 0) + i \Phi(0, \pi) \right] e^{i \theta_k}$$

$\Phi(0, 0) + i \Phi(\pi, \pi)$

$$\left[ \Delta_X(0, 0) - \Delta_Y(0, 0) \right] e^{-i \theta_k}$$

TR-broken:

$$\Delta_X(\pi, \pi) - \Delta_Y(\pi, \pi)$$

$$\left[ -\Delta_R(0, 0) + \Delta_X(0, 0) + \Delta_Y(0, 0) \right] e^{i \theta_k}$$
Surface Local Moment Regime

- The charge of $f$ electrons is localized
  - The $f$ orbital is half-filled $\Rightarrow$ Kondo lattice model
  - Light surface electrons?

- Kondo v.s. RKKY
  - Kondo wins: singlets frustrate the RKKY orders as doped “holes” $\Rightarrow$
    - spin liquid of localized $f$ electrons
    - ... charged uncondensed slave bosons + neutral $f$ spinons
    - ... fluctuating vortices in the slave boson phase
      $\Rightarrow$ 2D gauge flux for $d$ electrons (“QED3”)

- RKKY wins $\Rightarrow$
  - AF metal or insulator
  - VBS, spin liquid, etc. Dirac metal

Alexandrov, Coleman, Erten; Thomson, Sachdev
Conclusions

• Inter-orbital pairing in TIs
  – Spin-triplet pairing at large momenta due to S.O.C.
  – TR-invariant vortex lattice
  – Incompressible quantum (vortex) liquid
    (TR-invariant, non-Abelian)

• Possible realizations in:
  – Ultra-thin TI films
  – Kondo TI surfaces
Type-II Condensates

• Spin current by charge current + spin texture

\[
\eta = \begin{pmatrix} \eta_\uparrow \\ \eta_0 \\ \eta_\downarrow \end{pmatrix} = \begin{pmatrix} \zeta_+ e^{-i\theta} \\ \zeta_0 \\ \zeta_- e^{i\theta} \end{pmatrix} e^{i\gamma} \quad \eta = \begin{pmatrix} \eta_\uparrow \\ \eta_\downarrow \end{pmatrix} = \begin{pmatrix} \zeta_\uparrow e^{-\gamma/2} \\ \zeta_\downarrow e^{i\gamma/2} \end{pmatrix} e^{i\gamma}
\]

• Current densities & the Hamiltonian

\[\begin{align*}
\mathbf{j} & \propto \nabla \gamma \\
\mathbf{J}^x & \propto \cos \theta \mathbf{j} \\
\mathbf{J}^y & \propto \sin \theta \mathbf{j} \\
\mathbf{J}^z & \propto -\nabla \theta
\end{align*}\]

\[S \propto \hat{\theta}\]

\[\begin{align*}
\nabla \gamma & \sim \mathbf{j} \\
\hat{\theta} & \sim \mathbf{S}
\end{align*}\]

\[H = \int d^2r \left[ K (\nabla \gamma - \kappa \hat{\mathbf{z}} \times \hat{\theta})^2 + \frac{K'}{2} (\nabla \theta)^2 \right] \quad \text{Rashba S.O.C.} \Rightarrow \nabla \gamma \perp \hat{\theta}\]

Vortices and vortex states of Rashba spin-orbit coupled condensates
Type-II Vortices

\[ [D_\mu, I_\mu] = 0 \]

\[ \nabla^2 \theta \propto -2mv \hat{\theta} \nabla \gamma \to 0 \]
\[ \nabla^2 \gamma = 0 \]
\[ (\nabla \theta)(\nabla \gamma) \propto -mv (\hat{z} \times \nabla \theta) \hat{\theta} \]

- no sources for \( J^z \), and \( j \perp S \)
- no sources for \( j \)

- Vortex quadruplet
  - not classically (meta)stable
  - charge singularities bound to spin vortices (not antivortices)

\[ \nabla \theta \sim J^z \]
\[ \nabla \gamma \sim j \]
\[ \hat{\theta} \sim S \]
Localization of Hybridized Electrons

• Mott insulators in doped lattice models
  – Localization at \( p/q \) particles per lattice site
  – Broken translation symmetry

• Coulomb: may localize charge
  – Can't localize spin so easily on a TI surface!
  – S.O.C. \( \Rightarrow \) No spin back-scattering

• Compromise: spin liquid
  – Algebraic: Dirac points of spinons
  – Non-Abelian: fully gapped