The rise of topological quantum entanglement

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The rise of topological quantum entanglement
Overview

• Introduction to strongly correlated TIs
  – What we already (hopefully) know
  – Kondo TIs

• Correlations on Kondo TI boundaries
  – Predictions of new physics
  – Hybridized regime: SDW, SC, spin liquids...
  – Local moment regime: AF metal, QED3...

• Exotic quantum states in TI quantum wells
  – Novel vortex states
  – Fractional incompressible quantum liquids
Fundamental Questions

How to understand nature?

What is it made of?
(particle physics)

How is it organized?
(condensed matter physics)

Collective behavior of many particles

Emergent phenomena
(phases of matter)

Critical phenomena
(lose the sight of microscopics)
Correlations

By interactions between particles

Long-range order (spontaneous symmetry breaking)

By quantum entanglement

\[ |\text{Schrödinger's cat}\rangle = |\text{alive}\rangle + |\text{dead}\rangle \]

Electrons are prepared in a singlet spin state
total spin = 0 (which is conserved)

If either electron is measured with spin up = +1/2 (red arrow)
the other is always measured with its spin down = -1/2 (red)
The correlation is 100%.
Topological vs. Conventional

• Conventional states of matter
  – Local properties (order parameter) + symmetry determine all global properties

• Topological states of matter
  – Have global properties invisible to local probes (e.g. long-range entanglement)

• Examples of topological quantum states
  – Quantum Hall states & “topological insulators”
  – Spin liquids, string-net condensates...
Quantum Hall Effect

- **Integer QHE**
  - 2D topological band-insulator of electrons in magnetic field
  - Bulk spectrum: Landau levels (continuum) or Hofstadter (lattice)
  - Boundary spectrum: chiral gapless edge modes
  - Non-local property: Chern #, quantized transverse (Hall) conductivity
Quantum Hall Liquids

- **Fractional QHE**
  - Strongly correlated topological insulator of electrons in magn. field
  - Quasiparticles with fractional charge and exchange statistics
  - Ground-state degeneracy on a torus (no symmetry breaking)
  - Non-local property: many-body quantum entanglement (fract. stat.)
The Appeal of Topological

• Macroscopic quantum entanglement
  – What makes quantum mechanics fascinating...
  – Still uncharted class of quantum states

• Non-local properties are resilient
  – Standard for measurements of resistivity
  – Topological quantum computation

• The math of topology is fancy
TR-invariant topological Insulators

- Quantum (but not quantized) spin-Hall effect
  - Graphene
  - HgTe, Bi$_2$Se$_3$, Bi$_2$Te$_3$, etc. quantum wells

3D Topological Insulators

• Materials
  – Bi$_2$Se$_3$, Bi$_2$Te$_3$, etc.
  – SmB$_6$ and other Kondo insulators (?)

• TR symmetry
  – Protected “helical” Dirac metal on the crystal surface
  – Odd number of Dirac points

M.Z.Hasan, C.L.Kane, Rev.Mod.Phys. 82, 3045 (2010)
Topology + Symmetry

• Symmetry-protected boundary states
  – Charge conservation \(\Rightarrow\) edge states in QHE
  – TR symmetry \(\Rightarrow\) edge states in QSHE, Dirac metal in TIs
  – Crystal symmetry \(\Rightarrow\) additional Dirac points on TI boundaries

• Quantum anomalies
  – Local field theory on the boundary cannot be regularized
  \(\Rightarrow\) global entanglement on the boundary at cut-off scales (through the bulk)

Cancun, Mexico in June 2014
(proves that boundaries are most interesting)

Is our universe a topological boundary?
Kondo Insulators

• Materials:
  – SmB_6, YbB_{12}, Ce_3Bi_4Pt_3, Ce_3Pt_3Sb_3, CeNiSn, CeRhSb...

• Electron spectrum
  – Hybridized broad \(d\) and narrow \(f\) orbitals of the rare earth atom (e.g. Sm)
  – Heavy fermion insulators: \(E_f\) is inside the hybridization bandgap
  – Coulomb interactions \(\gg f\)-bandwidth

• Correlations
  – \(T\)-dependent gap
  – Collective modes
Kondo TIs (SmB$_6$)

- Is SmB$_6$ a topological insulator?
  - Theoretical proposal by band-inversions: Dzero, Galtski, Coleman
  - Residual $T\rightarrow 0$ resistivity & surface conduction
  - Quantum Hall effect
  - Latest neutron scattering
  - etc.

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A. Kebede, et al.
Physica B 223, 256 (1996)

D.J. Kim, J. Xia, Z. Fisk
arXiv:1307.0448

G. Li, et al.
arXiv:1306.5221
Collective modes in SmB$_6$

- Sharp peak in inelastic neutron scattering
  - Nearly flat dispersion at 14 meV
  - Protected from decay (inside the 19 meV bandgap)
  - Theory: perturbative slave boson model

Neutron scattering experiment: Broholm group at IQM/JHU
Theory: Fuhrman, PN
Collective Mode Theory

• Perturbative slave boson theory
  – Applied to the Anderson model adapted by Dzero, et al.
  – Input: renormalized band-structure + 3 phenomenological param.
  – Output: mode dispersion & spectral weight in the $1^{\text{st}}$ B.Z.

• Slave bosons
  – no double-occupancy of lattice sites by $f$ electrons

$$f_\alpha = b_\alpha^\dagger \psi_\alpha$$
$$\sum_\alpha \psi_\alpha^\dagger \psi_\alpha + b_\alpha^\dagger b_\alpha = 1$$

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SmB$_6$ Theory + Experiment

- Match the theoretical & experimental mode spectra
  - Fit the band-structure and phenomenological param.
  - deduce the renormalized fermion spectrum

**Implied band inversion at X ⇒ a TI**
Kondo TI Boundary

- A 2D “helical” heavy fermion system

SmB₆ metallic surface electron dispersion

The phase diagram of heavy fermion “metals”

Bands “bend” near the boundary

Hybridized regime

Local moment regime

Details are not universal

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Hybridized Surface Regime

• 2D perturbative slave boson theory
  – Slave bosons mediate forces among electrons
  – attractive in exciton, repulsive in Cooper ch.

Basic process:

\[ f-d \] Hybridization assisted by a slave boson evolves into Kondo singlet upon localization

\[ \tilde{\Gamma}(q) = \begin{array}{c}
\text{slave boson renormalization}
\end{array} \]

\[ \tilde{\Gamma}'(q) = \begin{array}{c}
\text{slave boson renormalization}
\end{array} \]

• Instabilities by nesting
  – Magnetic (SDW) \( \Rightarrow \) Dirac points perish
  – Superconductivity (s\( ^\pm \) or d-wave) \( \Rightarrow \) Dirac points live

Recall: we have a collective mode right at these wavevectors

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Localization of Hybridized Electrons

- Mott insulators in doped lattice models
  - Localization at $p/q$ particles per lattice site
  - Broken translation symmetry

- Coulomb: may localize charge
  - Can't localize spin so easily on a TI surface!
  - S.O.C. $\Rightarrow$ No spin back-scattering

- Compromise: spin liquid
  - Algebraic: Dirac points of spinons
  - Non-Abelian: fully gapped

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Surface Local Moment Regime

• The charge of $f$ electrons is localized
  – The $f$ orbital is half-filled
  – Low-energy dynamics: Kondo lattice model

• Kondo v.s. RKKY
  – RKKY wins ⇒
    • AF metal or insulator (gapped Dirac points)
    • VBS, spin liquid, etc. Dirac metal
  – Kondo wins (unrealistic over-screening) ⇒
    • featureless conventional Dirac metal
  – Kondo singlets frustrate the RKKY orders as doped “holes” ⇒
    • spin liquid of $f$ electrons + 2D QED of conduction electrons
Correlated TI quantum wells

- Perturb a QCP by the spin-orbit effect
  - New phases emerge due to relevant scales

\[ \tau^z = +1 \quad \phi_+ \]
\[ \tau^z = -1 \quad \phi_0 \]

PN, T.Duric, Z.Tesanovic
PRL 110, 176804 (2013)

PN, Z.Tesanovic
PRB 87, 104514 (2013)

PN, Z.Tesanovic
PRB 87, 134511 (2013)
Competing orders

- Competition for 1% of free energy ($F$)
  - PDW map (by ordering wavevectors):
    - Commensurate pair density waves
    - Vortex lattices
    - Incommensurate plane waves

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Helical spin currents on a lattice

- Main competitors for the ground state:
  - pair density wave (PDW) states usually win, but found only when looked for!
  - SU(2) vortex lattices (type-I structures) always found in unconstrained minimization, sometimes win?

Similar to:
Vortex lattice melting

• SU(2) vortex lattice
  – Array of “chiral” vortices and antivortices
  – Ideally one vortex per Yang-Mills “flux quantum”
  – Frustrated by the crystal lattice

• Quantum fluctuations
  – Positional fluctuations of vortices
  – Grow when the condensate is weakened by tuning the gate voltage
  – Eventually, 1\textsuperscript{st} order phase transition (preempts the 2\textsuperscript{nd} order one)

• Vortex liquid
  – Particles per flux quantum $\sim$ 1
  – Fractional TI?

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Rashba S.O.C. on a TI's Boundary

- Particles + static SU(2) gauge field

\[
H = \frac{(\mathbf{p} - g \mathbf{A})^2}{2m} + \ldots
\]

SU(2) generators (spin projection matrices)

\[
\mathbf{A}_\mu = (\mathbf{A}, A_0) = A^a_\mu S^a
\]

Yang-Mills flux matrix (“magnetic” for \(\mu = 0\))

\[
\Phi^\mu = \epsilon^{\mu\nu\lambda} (\partial_\nu A_\lambda - ig A_\nu A_\lambda)
\]

D. Hsieh, et.al, PRL 103, 146401 (2009)
Y. Zhang, et.al, Nature Phys. 6, 584 (2010)

- Rashba S.O.C. ⇒ Dirac spectrum

\[
\mathbf{A} = -mv(\hat{z} \times \mathbf{S}) \quad \ldots \quad \Phi^0 = g(mv)^2 S^z
\]

\[
H = \frac{p^2}{2m} + vg\hat{z}(\mathbf{S} \times \mathbf{p}) + \text{const.}
\]

Cyclotron: \(\omega_\Phi = \Phi_0^z/m \gg 100 \text{ meV}\)
SU(2) flux: \(n_\Phi = \Phi_0^z/\hbar^2 \gg 2 \cdot 10^{15} \text{ m}^{-2}\)
Effective theory of fractional TIs

- **Landau-Ginzburg theory (dual Lagrangian of vortices)**

  \[
  \mathcal{L}_{\text{LG}} = \frac{K}{2} \left| (\partial_{\mu} - i B_{\mu}) \psi \right|^2 - t |\psi|^2 - t' \psi^\dagger \Phi_0 \psi + u |\psi|^4 + v |\psi^\dagger \gamma^a \psi|^2 + v' |\psi^\dagger \Phi_0 \psi|^2 \\
  + \frac{1}{8\pi^2} \text{tr} \left[ Q^{-2} (\Phi_B - \Theta_{\mu})^2 \right]
  \]

  - Maxwell term (density fluctuations)

- **Topological term**

  - equation of motion \(\Rightarrow\) “drift currents” in \(U(1) \times SU(2)\) E&M field

  \[
  \mathcal{L}_t = \frac{i}{8} \psi^\dagger \epsilon^{\mu\nu\lambda} \left[ D_{\mu} \{ D_{\nu}, \Phi_0 \} D_{\lambda} + \{ D_{\mu} D_{\nu} D_{\lambda}, \Phi_0 \} \right] \psi
  \]

  - Levi-Civita tensor
  - anti-commutators
  - flux matrix

  \[D_{\mu} = \partial_{\mu} - i A_{\mu}\]

  \[\Phi^\mu = \epsilon^{\mu\nu\lambda} (\partial_{\nu} A_{\lambda} - i A_{\nu} A_{\lambda})\]

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Duality

• Boson-vortex duality in (2+1)D

\[ \omega_c \]

incompressible quantum liquids
(dominant flux "cyclotron" scale)

superconductor
(\( \xi \) particle condensate)

Mott insulator
(\( \psi \) vortex condensate)

QCP

\[ V_G \]

Exact if:
– particles are bosons
– \( S^z \) is conserved

Conjecture:
– generalize by symmetry
– fermionic particles?

• Particle Lagrangian

\[ \mathcal{L}_{\text{LG}} = \xi^{\dagger} (\partial_0 - iA_0) \xi + \left[ (\partial_i - iA_i) \xi \right]^{\dagger} \frac{\tilde{K}}{2} \left[ (\partial_i - iA_i) \xi \right] - \xi^{\dagger} \tilde{\psi} \xi + \tilde{u} |\xi|^4 + \tilde{v} |\xi^{\dagger} \gamma^a \xi|^2 \]

\[ \mathcal{L}_{\text{PT}} = \frac{i}{8} \xi^{\dagger} \epsilon^{\mu\nu\lambda} \left[ \partial_\mu \left\{ \partial_\nu, \Theta^{-1} \right\} \partial_\lambda + \left\{ \partial_\mu \partial_\nu \partial_\lambda, \Theta^{-1} \right\} \right] \xi \]
Incompressible Quantum Liquids

• Quantum (spin) Hall:

\[ \xi(r) = \begin{pmatrix} C_\uparrow e^{i\varphi_\uparrow(r)} \\ C_\downarrow e^{i\varphi_\downarrow(r)} \end{pmatrix}, \quad \psi(r) = \begin{pmatrix} \sqrt{\rho_\uparrow} e^{i\theta_\uparrow(r)} \\ \sqrt{\rho_\downarrow} e^{i\theta_\downarrow(r)} \end{pmatrix} \]

\[ L_{CS} = \sum_s \left[ \frac{i\sigma_s}{4\pi m} \varepsilon^{\mu\nu\lambda} b_{s\mu} \partial_\nu b_{s\lambda} - i \frac{e^{\mu\nu\lambda}}{4\pi m} b_{s\mu} \partial_\nu A_\lambda^z + \frac{1}{8\pi^2 a^2} (\varepsilon^{\mu\nu\lambda} \partial_\nu b_{s\lambda} - \Theta s_0 \delta_{0\lambda})^2 \right] \]

Chern-Simons \hspace{2cm} Maxwell

• Rashba:

\[ \xi_\pm(p) = \frac{f_\pm(p)}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm e^{i\phi_p} \end{pmatrix} \quad \phi_p = \text{arg}(-p_y + ip_x) \]

\[ L_p = \frac{i}{8} \xi^\dagger \varepsilon^{\mu\nu\lambda} \left[ \partial_\mu \{ \partial_\nu, \Theta_0^{-1} \} \partial_\lambda + \{ \partial_\mu \partial_\nu, \partial_\lambda, \Theta_0^{-1} \} \right] \xi \rightarrow -\frac{i}{8} \text{tr} \left[ \{ Z_\mu, \Theta_0^{-1} \} \{ \Phi_Z^\mu, \xi \xi^\dagger \} \right] \]

effective Chern-Simons theory w.
constrained non-Abelian gauge field
Conclusions

• Neutron studies of SmB$_6$
  – Clear evidence of strong correlations
  – Points to a topological band-structure

• Kondo TI boundaries
  – Correlated states at the crystal boundary
  – Hybridized regime: Dirac metal, SDW, SC, spin liquids...
  – Local moment regime: AF metal/insulator, helical 2D QED

• Kondo (and other) TI quantum wells
  – SU(2) vortex lattice, non-Abelian fractional TIs

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