Unitarity in lattices and Cooper pair insulators

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Acknowledgments

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Overview

- Introduction and motivation
- Unitarity in lattice potentials (renormalization group)
- Cooper pair insulators (“pseudogap” at $T=0$)
- Applications: re-entrant SC, PDW, topological insulators, cuprates...
- Conclusions
Introduction and motivation

- The quest for universality
  - Common phenomena independent of complicated microscopic details
  - The fundamental principle behind understanding things

- Universality in fermionic systems
  - In metals: Fermi surface instabilities (not a subject of this talk)
  - In band insulators: scattering resonances and pairing ... unitarity
  - In correlated states with emergent degrees of freedom

- Can unitarity teach us about:
  - The birth of some correlated states in atomic and electronic materials?
  - “Unconventional” superconducting transitions?
Universality from band insulators

- Gapped interacting fermionic excitations
  - A direct path to correlated states in which...
  - … low-energy bosons dominate dynamics

The simplest setup
- A band insulator with short-range interactions
- Weak: BCS pairing transitions
- Strong: “unconventional” transitions in bosonic universality classes
- Fixed points describe scattering resonances

Applicability
- Cold atoms in lattice potentials
- PDW and topological insulators
- High-temperature superconductors?
Quantum vortex liquid in cuprates?

- Vortices in the normal phase of cuprates, even at $T=0$


La$_{2-x}$Sr$_x$CuO$_4$

$T_{\text{onset}}$, $T^*$


Vortices in superconductors

- “Fluctuating” $d$-wave superconductivity
  - Massless Dirac fermions
  - $d$-wave $\rightarrow$ no vortex core states
  - Small cores
  - Light and friction-free vortices
  - Quantum vortex dynamics

- Conventional BCS superconductivity
  - $s$-wave $\rightarrow$ vortex core states
  - Large cores
  - Heavy vortices, large friction
  - Semi-classical vortex dynamics

Unitarity: two-body picture

- Universality: irrelevant microscopic details
- Two-body resonant scattering
- Bound state at zero energy

\[ \sigma \approx 4\pi a^2 \quad , \quad ka \ll 1 \]

\[ \sigma \approx 4\pi a^2 \left(1 - \frac{\tan(\alpha_0 a)}{\alpha_0 a}\right) \quad , \quad ka \ll 1 \]

\[ \sigma \approx \frac{4\pi}{k^2 + \alpha_0^2 \cot(\alpha_0 a)} \approx \frac{4\pi}{k^2} \quad , \quad \alpha_0 = \sqrt{2mV_0} \]
Unitarity: many-body picture

\[ S = \int d\tau d^d x \left[ \psi_{i\alpha}^\dagger \left( \frac{\partial}{\partial \tau} + \frac{(-i \nabla - A)^2}{2m} - \mu + V(x) \right) \psi_{i\alpha} ight. \\
+ h \left( \psi_{i\uparrow}^\dagger \psi_{i\uparrow} - \psi_{i\downarrow}^\dagger \psi_{i\downarrow} \right) + N \frac{m \nu}{4\pi} \Phi^\dagger \Phi + \Phi^\dagger \psi_{i\downarrow} \psi_{i\uparrow} + \Phi \psi_{i\uparrow}^\dagger \psi_{i\downarrow}^\dagger \left. \right] \]

- Universality
- Quantum critical point
- Zero density, $T=0$

New fixed points in lattice potentials

- Unitarity at finite densities
  - Every band insulator is a vacuum of particles and holes

- SF-I pairing transitions
  - (p) … particle dominated
  - (h) … hole dominated
  - (ph) .. relativistic

- Tuning parameters
  - Chemical potential $\mu$
  - Interaction strength $\nu$
  - Lattice depth $V$
Transitions involving only particles (holes)

- **Effective action**

\[
S = \int \mathcal{D}k \ f_{k,\alpha}^{\dagger} (-i\omega + E(k)) f_{k,\alpha} + U \int \mathcal{D}k_1 \mathcal{D}k_2 \mathcal{D}q \ f_{k_1,\alpha}^{\dagger} f_{k_1+q,\alpha} f_{k_2,\beta}^{\dagger} f_{k_2-q,\beta}
\]

\[
E(k) = E_0 + \frac{k^2}{2m} \quad E_g = E_0 + U
\]

- **Exact renormalization group**

\[
\frac{dE_g}{dl} = 2E_g \quad , \quad \ldots \quad = 0
\]

\[
\frac{dU}{dl} = (2 - d)U - \Pi U^2 + \ldots
\]
Transitions involving only particles (holes)

- Fixed points & RG flow
  - $d<2$: Gaussian & Tonks-Girardeau
  - $d=2$: “Gaussian”
  - $d>2$: Gaussian & Unitarity

- Run-away flow for $U<U^*<0$
  - Asymptote at a finite $l$
  - High-energy pairing
    - bound-state pairs
    - BEC regime

- BCS: $U^*<U<0$

\[
U(l) = \begin{cases} 
\frac{U(0)}{1 + \Pi U(0)} & , \quad d = 2 \\
\frac{U(0)}{[1 + \Pi U(0)]e^l - \Pi U(0)} & , \quad d = 3 
\end{cases}
\]
Transitions involving both particles and holes

Effective action

\[ S = \sum_n \int \mathcal{D}k \ f_{n,k,\alpha}^\dagger (-i\omega + E_n(k)) f_{n,k,\alpha} \]

\[ + \sum_{n_1 m_1} \sum_{n_2 m_2} U_{n_1 n_2}^{m_1 m_2} \int \mathcal{D}k_1 \mathcal{D}k_2 \mathcal{D}q \ f_{m_1,k_1+q,\alpha}^\dagger f_{n_1,k_1,\alpha} f_{m_2,k_2-q,\beta}^\dagger f_{n_2,k_2,\beta} \]

\[ E_c(k) = E_{c0} + \frac{k^2}{2m_c} \]

\( n = c \ldots \text{conduction band} \)

\[ E_v(k) = -E_{v0} - \frac{k^2}{2m_v} \]

\( n = v \ldots \text{valence band} \)

\[ + \text{scattering with one band-conversion ...} \]
Transitions involving both particles and holes

- Renormalization group: one loop, $\varepsilon = d-2$ expansion

- Band mixing
  - Generated by scattering with one band-conversion

\[
\begin{align*}
\text{Unitarity in lattices and Cooper pair insulators} & \\
(14/34)
\end{align*}
\]
Transitions involving both particles and holes

- RG equations with no band-mixing: admit analytical solution

\[
\frac{du_c}{dl} = \epsilon \left[-u_c - 4u_c^2 - 4u_e^2\right]
\]

\[
\frac{du_v}{dl} = \epsilon \left[-u_v - 4u_v^2 - 4u_e^2\right]
\]

\[
\frac{du_{cv}}{dl} = \epsilon \left[-u_{cv} + 2u_{cv}^2 + 8(1 - \beta^2)u_e^2\right]
\]

\[
\frac{du_m}{dl} = \epsilon \left[-u_m - 4u_m^2 + 4u_{cv}u_m\right]
\]

\[
\frac{du_e}{dl} = \epsilon \left[-u_e + u_e\left(-4u_c - 4u_v + 8u_{cv} - 4u_m\right)\right]
\]

\[
\frac{de_g}{dl} = 2e_g - \frac{2u_v}{1 - \beta} + \frac{2u_{cv}}{1 - \beta^2} - \frac{u_m}{1 - \beta^2}
\]

Unitarity in lattices and Cooper pair insulators
Transitions involving both particles and holes

- 17 fixed points with no band-mixing
  - Gaussian + 15 resonant scattering fixed points in various channels
  - A “pair-scattering” fixed point

Unitarity in lattices and Cooper pair insulators
Transitions involving both particles and holes

- Scattering resonances
  - BEC-BCS crossovers in various channels

Bound singlet of two conduction band or two valence band fermion

Bound singlet of a conduction and a valence band fermion

Exciton: bound singlet of a particle and hole

Assisted resonance?

Extended s-wave resonating singlet
RG summary

- Most general cases
  - Scattering resonances with multiple particle/hole species
  - Additional fixed points with band-mixing

- Unitarity universality classes
  - Universal ratios of observables
    - $(\mu, E_f, T_c, P ...)$
  - Slight modifications of the vacuum universality

- Global RG
  - Run-away flows in BEC limits
  - Strong-coupling fixed points

P.N., Arxiv:1006.2378 (2010)
Effective BEC regimes

- **RG run-away flow (bound states)**
  - Any interaction strength in $d=2$
  - Requires strong interaction in $d>2$

- **BEC regime: bound states**
  - Quasiparticles are gapped
  - Bosonic universality class for the SF transition
    mean-field or XY

- **BCS regime: no bound states**
  - BCS (pairing) transition
  - Must close the fermion gap in order to induce SF
Bosonic Mott Insulator

- Insulator adjacent to SF
  - BEC: Bosonic Mott
  - BCS: Band insulator

Bosonic Mott insulator
- No symmetry breaking (but not excluded either)
- Bosonic lowest energy excitations
- Large fermion gap

P.N., Zlatko Tešanović (unpublished)
Mott/band insulator distinction

- Quantum phase transition
  - Non-analytic change of the ground state manifold as a function of tuning parameters

- A generalization to the entire spectrum
  - Non-analytic change of the Hamiltonian (density matrix) as a function of tuning parameters

- Mott insulator “order parameter”

\[
\rho(E, \mathbf{P}) = \frac{1}{\mathcal{V}} \sum_n \delta(E - E_n)\delta(\mathbf{P} - \mathbf{P}_n)
\]

\[
\rho'(\mathbf{k}) = \lim_{\Delta \rightarrow 0} \lim_{\Delta_{p} \rightarrow 0} \int d\varepsilon \int d^d p \frac{1}{\mathcal{V}} \sum N \rho(N \varepsilon_k + \delta \varepsilon, N \mathbf{p}_k + \delta \mathbf{p})
\]
Non-equilibrium pairing transitions

- Cooper pair laser
  - Sharp non-equilibrium distinction between Mott and band insulators

- Numerics?
  - Quantum Monte Carlo
  - Negative-$U$ Hubbard model

- Experiments?
  - SC / narrow bandgap material heterostructures
  - Superlattices
  - Cold atoms

P.N., Zlatko Tešanović (unpublished)
Non-trivial Mott insulators

- **Broken symmetries**
  - Pair density waves (particle-particle BEC)
  - Magnetic, nematic... ($p$-wave pairs?)
  - Density waves (particle-hole BEC)
  - Valence bond crystals (inter-valley particle-particle BEC)

- **Topological orders**
  - Fractional quantum Hall states with even-denominator filling factor
  - Fractional spin quantum Hall states?
  - Spin liquids
Superfluids in the quantum Hall regime

- Normal state \(\rightarrow\) quantum Hall insulator
  - Localized particles (cyclotron orbitals)
  - Discrete Landau levels
  - Macroscopic degeneracy: two particles per flux quantum

**Superfluid**

\[
\Phi((r)) = \Delta_0 e^{-2m\omega y^2} \theta_3 \left( \left( \pi \sqrt{3} m \omega \right)^{\frac{1}{2}} (x + iy) \right) e^{i\pi/3}
\]
No $p_x$ dependence to all orders of $1/N$

- “charged” bosonic excitations live on degenerate Landau levels
- Macroscopically many modes turn soft simultaneously
- The nature of “condensate” is determined by interactions
Quantum vortex lattice melting

- **Vortex mass**
  - Compression of the stiff superfluid
  - Neutral: 
    \[ m_v \approx \frac{\rho_s}{s^2} \log \left( \frac{R}{\xi} \right) \]
    \[ \rho_s, s^2 \propto |\Phi|^2 \]

- **Vortex localization energy**
  - \[ E_{\text{kin}} \sim p^2/2m_v \quad \ldots \quad p^2 \sim B \]

- **Vortex lattice potential energy**
  - \( \Pi \) is degenerate \( \rightarrow E_{\text{pt}} \sim \Phi_0^4 \)

\[
\frac{\mathcal{F}(\Phi_0)}{N} = \frac{\mathcal{F}_0}{N} + \hat{\Pi}_{ij} \Phi_0^i \Phi_0^j + \hat{U}_{ijkl} \Phi_0^i \Phi_0^j \Phi_0^k \Phi_0^l + \mathcal{O}(\Phi^6)
\]

Unitarity in lattices and Cooper pair insulators
Vortex liquid

- Genuine phases at $T=0$
  - Vortex lattice potential energy: $\Delta_0^4$
  - Melting kinetic energy gain: $\log^{-1}(\Delta_0)$
  - 1st order vortex lattice melting as $\Delta_0 \to 0$
  - Low energy spectrum inconsistent with fermionic quantum Hall states
  - Non-universal properties (by RG)


Unitarity in lattices and Cooper pair insulators
The nature of vortex liquids

Non-universal properties
- At Gaussian and unitarity fixed points of RG

\[ S = \int d\tau d^{d-2}r_\perp \left\{ \sum_n \int \frac{dk_x}{2\pi} \psi_{n,k_x}^\dagger \left( \frac{\partial}{\partial \tau} + n\omega_c - \frac{\nabla^2}{2m} - \mu' \right) \psi_{n,k_x} + N \sum_{n_1n_2} \int \frac{dp_x}{2\pi} \Phi_{n_1,p_x}^\dagger \hat{\Pi}_{n_1,n_2}^{(0)} \Phi_{n_2,p_x} \right. \]

+ \left. g \sum_{nm_1m_2} \int \frac{dk_x dp_x}{2\pi} \frac{1}{\sqrt{B}} \Gamma_{m_1m_2}^n \left( \frac{k_x}{2} \right) \left[ \Phi_{n,p_x}^\dagger \psi_{m_1,k_x+\frac{p_x}{2}} \psi_{m_2,-k_x+\frac{p_x}{2}} + \text{h.c.} \right] \right. 

+ u_2 \sum_{m_1 \ldots m_4} \int \frac{dk_{x1} dk_{x2} dq_x}{2\pi} \frac{1}{2\pi} \Gamma_{m_1 \ldots m_4}^{m'} \left( k_{x1}, k_{x2}, q_x \right) \psi_{m_1,k_{x1}}^\dagger \psi_{m_2,k_{x2}}^\dagger \psi_{m_3,k_{x2}+q_x} \psi_{m_4,k_{x1}-q_x} \right\} + \cdots

- All interactions are relevant in \( d=2 \)
- Dimensional reduction
- Many stable interacting fixed points?

\[ \frac{dg}{dl} = \left( 3 - \frac{d}{2} \right) g - bNg^3 \]

\[ \frac{du_n}{dl} = \left[ d + (2 - d)n \right] u_n + \mathcal{O}(u^2) \]

BCS-BEC crossover in lattice potentials

- 2nd order superfluid-insulator phase transition at $T=0, \ h=0$
- Band-Mott insulator crossover at unitarity (s-wave)

Pair density wave

- Supersolid without the uniform component
- Pairing instability in a band-insulator **generally** occurs at a finite crystal momentum

\[
\Pi_{Gq;G'q'} = \sum_{n_1 n_2} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \frac{f(\xi_{n_1 k_1}) - f(-\xi_{n_2 k_2})}{\xi_{n_1 k_1} + \xi_{n_2 k_2}} \Gamma_{n_1 k_1; n_2 k_2} \Gamma^*_{Gq; G'q'}
\]
**PDW evolution**

- **Incommensurrate PDW**
  - Vertex $q$-dependence
  - Weak coupling (BCS limit)

- **Commensurate PDW**
  - Energy $q$-dependence
  - Strong inter-band coupling
  - Halperin-Rice in p-p

Incommensurate supersolid?
- Pairing bubble has non-analytic linear $q$-dependence at small $q$
- Inconsistent with $q=0$ pairing ($\omega \sim \sqrt{|q|}$ Goldstone modes)
- Robust finite-$q$ pairing against fluctuations
- But, frustrated on the lattice!

Fluctuation effects
- Stabilize a commensurate supersolid order?
- Looks like Mott physics!
- Are there non-trivial paired insulators?

Near the superfluid-insulator transition
- Fermions have a large (band) gap
- Collective bosonic modes are low energy excitations
- Charge conservation $\Rightarrow$ infinite lifetime for gapped bosons
Cuprates, d-wave pairing and Mottness

- Microscopic mechanism in underdoped cuprates?
  - Short-range AF correlations $\Rightarrow$ gap (antinodal)
  - Hole pair hopping doesn't frustrate spins
    $\Rightarrow$ effective weak attractive interaction (antinodal)
  - Two-dimensional dynamics
  - Effective BEC regime for antinodal quasiparticles

- Consequences
  - Mottness adjacent to SC, quantum vortex dynamics...

- Complications due to d-wave
  - Nodal pairbreaking occurs, but anomalously slow
  - Low-energy bosons exist (superohmic decay) with large DOS
Conclusions

- Effective scattering resonances
  - “Weak-coupling” universality in generic band insulators
  - Particle-particle and particle-hole channels
  - Path to strongly correlated states of fermions

- Bosonic Mott insulators from fermions
  - Adjacent to superfluid phases in effective BEC regimes (especially 2D)
  - Susceptible to symmetry breaking or topological order

- Systems of interest
  - PDW in cold atom gases
  - Re-entrant superconductivity, topological insulators
  - Cuprates