Pairing Instability in 2D Rotating Fermion Liquids Near Unitarity

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Motivation

- Vortex liquids
  - What are they?
  - How to obtain them?

- Re-entrant superconductivity
  - SC above $H_{c2}$

- FFLO states
  - Orbital+Zeeman effect

- Ultra-Cold atoms
  - Vortices & vortex lattices
  - Quantum Hall physics
  - Strong correlations near unitarity

Perturbation theory: $1/N$ expansion

\[
S = \int d\tau d^d x \left[ \psi_{i\alpha}^\dagger \left( \frac{\partial}{\partial \tau} + \frac{(-i \nabla - A)^2}{2m} - \mu + V(x) \right) \psi_{i\alpha} 
+ \hbar (\psi_{i\uparrow}^\dagger \psi_{i\uparrow} - \psi_{i\downarrow}^\dagger \psi_{i\downarrow}) + N \frac{m\nu}{4\pi} \Phi^\dagger \Phi + \Phi^\dagger \psi_{i\downarrow} \psi_{i\uparrow} + \Phi \psi_{i\uparrow}^\dagger \psi_{i\downarrow}^\dagger \right]
\]

\[\nabla \times A = B = \tilde{z} m \omega_c = \tilde{z} 2m \omega\]

- Feynman diagrams

  - Physical atom (fermion)
  - Cooper pair, molecule (boson)
  - Vertex
Pairing instability

\[
\Pi_{n,n'}(p_x, p_z, i\Omega) \propto \sum_{m_1,m_2} \int \frac{dk_z}{2\pi} \frac{dk_x}{2\pi} \frac{f(\varepsilon_{m_1,k_z+p_z/2}) - f(-\varepsilon_{m_2,-k_z+p_z/2})}{-i\Omega + \varepsilon_{m_1,k_z+p_z/2} + \varepsilon_{m_2,-k_z+p_z/2}} \times \\
\times \Gamma_{m_1,m_2}^n \left( \frac{k_x}{\sqrt{B}} \right) \Gamma_{m_1,m_2}^{n'*} \left( \frac{k_x}{\sqrt{B}} \right) + O \left( \frac{1}{N} \right)
\]

- No \( p_x \) dependence at all orders of \( 1/N \)
- “charged” bosonic excitations live on degenerate Landau levels
- Macroscopically many modes turn soft simultaneously
- The nature of “condensate” is determined by interactions
Pairing instability

- Quantum Hall $\to$ superfluid
- $2^{\text{nd}}$ order (saddle-point)

Quantum vortex lattice melting

- Vortex mass
  - Compression of the stiff superfluid
  - Neutral: \( m_v \approx \frac{\rho_s}{s^2} \log \left( \frac{R}{\xi} \right) \)
    \( \rho_s, s^2 \propto |\Phi|^2 \)

- Vortex localization energy
  - \( E_{\text{kin}} \sim p^2/2m_v \quad \ldots \quad p^2 \sim B \)

- Vortex lattice potential energy
  - \( \Pi \) is degenerate \( \rightarrow E_{\text{pot}} \sim |\Phi_0|^4 \)

\[
\frac{\mathcal{F}(\Phi_0)}{N} = \frac{\mathcal{F}_0}{N} + \hat{\Pi}_{ij} \Phi_0^i \Phi_0^j + \hat{U}_{ijkl} \Phi_0^i \Phi_0^j \Phi_0^k \Phi_0^l \right| O(\Phi^6)
\]
Vortex lattice melting

- Vortex lattice melts at finite $\Phi_0$ as $\Phi_0 \rightarrow 0$
  - 1st order by Landau-Ginzburg

- Vortex liquid ground-state
  - Superposition of many vortex arrangements
  - Result of some “mixing” perturbation
  - Energy gain $\rightarrow$ low energy bosonic levels split
  - Different structure (degeneracy) than in the fermionic quantum Hall states
  - A genuine quantum phase
Vortex liquid

- Genuine phase at $T=0$
  - Strongly correlated insulator
  - Cooper pairs are formed
  - Possibly destroyed at finite $T$

- Examples
  - Quantum Hall state $\nu=1/2$
  - Wigner crystal of Cooper pairs

The nature of vortex liquids

- Non-universal properties
  - At Gaussian and unitarity fixed points of RG

\[
S = \int d\tau d^{d-2}r_\perp \left\{ \sum_n \int \frac{dk_x}{2\pi} \psi_{n,k_x}^\dagger \left( \frac{\partial}{\partial \tau} + n\omega_c - \frac{\nabla^2}{2m} - \mu' \right) \psi_{n,k_x} + N \sum_{n_1n_2} \int \frac{dp_x}{2\pi} \Phi_{n_1,p_x}^\dagger \hat{\Pi}_{n_1,n_2}^{(0)} \Phi_{n_2,p_x} \right. \\
+ g \sum_{nm_1m_2} \int \frac{dk_x dp_x}{2\pi} \frac{\Gamma^n}{m_1m_2} \left( \frac{k_x}{\sqrt{B}} \right) \left[ \Phi_{n,p_x}^\dagger \psi_{m_1,k_x+p_x/2} \psi_{m_2,-k_x+p_x/2} + \text{h.c.} \right] \\
+ u_2 \sum_{m_1\ldots m_4} \int \frac{dk_{x1} dk_{x2} dq_x}{2\pi} \frac{\Gamma'}{m_1\ldots m_4} \left( k_{x1}, k_{x2}, q_x \right) \psi_{m_1,k_{x1}}^\dagger \psi_{m_2,k_{x2}}^\dagger \psi_{m_3,k_{x2}+q_x} \psi_{m_4,k_{x1}-q_x} \left\} + \cdots 
\]

- All interactions are relevant in \( d=2 \)
  - Dimensional reduction
  - Many stable interacting fixed points?

\[
\frac{dg}{dl} = \left( 3 - \frac{d}{2} \right) g - bNg^3 \\
\frac{du_n}{dl} = \left[ d + (2-d)n \right] u_n + \mathcal{O}(u^2)
\]
Competing forces
- Pairing, orbital, Zeeman
- FFLO-metals and FFLO-insulators

Strongly Correlated Physics With Ultra-Cold Atoms
Conclusions

- Rotating fermions near unitarity
  - Novel strongly correlated physics
  - Connections with cuprates

- Re-entrant superconductivity
  - Pairing in low Landau levels

- Vortex liquids
  - Strongly correlated quantum insulators of Cooper pairs
Perturbation theory: $1/N$ expansion

- Full bosonic propagator (Dyson equation)
  \[
  \begin{array}{c}
  \begin{array}{c}
  \quad = \frac{1}{N} \left[ (\quad)^{-1} + \quad \right]^{-1}
  \\
  \quad = \quad + \quad + \quad + \quad + \quad + \ldots
  \\
  \end{array}
  \\
  \quad = \quad + \quad + \quad + \quad + \quad + \ldots
  \\
  \end{array}
  \]

- No natural small parameter
  - Semi-classical expansion: $N=\infty$ is mean-field approximation
  - Physical: $N=1$