Excerpts from Galileo Galilei
Dialogues Concerning Two New Sciences [1638]

THIRD DAY: CHANGE OF POSITION. [De Motu Locali]

INTRODUCTION   MY purpose is to set forth a very new science dealing
with a very ancient subject. There is, in nature, perhaps nothing older than
motion, concerning which the books written by philosophers are neither few
nor small; nevertheless I have discovered by experiment some properties of it
which are worth knowing and which have not hitherto been either observed or
demonstrated. Some superficial observations have been made, as, for instance,
that the free motion [naturalem motum] of a heavy falling body is continuously
accelerated [Natural motion of the author has here been translated into free
motionsince this is the term used to-day to distinguish the natural from the
violent motions of the Renaissance]; but to just what extent this acceleration
occurs has not yet been announced; for so far as I know, no one has yet pointed
out that the distances traversed, during equal intervals of time, by a body falling
from rest, stand to one another in the same ratio as the odd numbers beginning
with unity [A theorem demonstarted in the text].

It has been observed that missiles and projectiles describe a curved path of
some sort; however no one has pointed out the fact that this path is a parabola.
But this and other facts, not few in number or less worth knowing, I have
succeeded in proving; and what I consider more important, there have been
opened up to this vast and most excellent science, of which my work is merely
the beginning, ways and means by which other minds more acute than mine
will explore its remote corners.

This discussion is divided into three parts; the first part deals with motion
which is steady or uniform; the second treats of motion as we find it accelerated
in nature; the third deals with the so-called violent motions and with projectiles.

UNIFORM MOTION

Definition: By steady or uniform motion, I mean one in which the dis-
tances traversed by the moving particle during any equal intervals of time, are
themselves equal.

NATURALLY ACCELERATED MOTION   The properties belonging to
uniform motion have been discussed in the preceding section; but accelerated
motion remains to be considered.

And first of all it seems desirable to find and explain a definition best fitting
natural phenomena. For anyone may invent an arbitrary type of motion and
discuss its properties; thus, for instance, some have imagined helices and con-
choids as described by certain motions which are not met with in nature, and
have very commendably established the properties which these curves possess
in virtue of their definitions; but we have decided to consider the phenomena of

bodies falling with an acceleration such as actually occurs in nature and to make this definition of accelerated motion exhibit the essential features of observed accelerated motions. And this, at last, after repeated efforts we trust we have succeeded in doing. In this belief we are confirmed mainly by the consideration that experimental results are seen to agree with and exactly correspond with those properties which have been, one after another, demonstrated by us. Finally, in the investigation of naturally accelerated motion we were led, by hand as it were, in following the habit and custom of [161] nature herself, in all her various other processes, to employ only those means which are most common, simple and easy.

For I think no one believes that swimming or flying can be accomplished in a manner simpler or easier than that instinctively employed by fishes and birds.

When, therefore, I observe a stone initially at rest falling from an elevated position and continually acquiring new increments of speed, why should I not believe that such increases take place in a manner which is exceedingly simple and rather obvious to everybody? If now we examine the matter carefully we find no addition or increment more simple than that which repeats itself always in the same manner. This we readily understand when we consider the intimate relationship between time and motion; for just as uniformity of motion is defined by and conceived through equal times and equal spaces (thus we call a motion uniform when equal distances are traversed during equal time-intervals), so also we may, in a similar manner, through equal time-intervals, conceive additions of speed as taking place without complication; thus we may picture to our[198] mind a motion as uniformly and continuously accelerated when, during any equal intervals of time whatever, equal increments of speed are given to it. Thus if any equal intervals of time whatever have elapsed, counting from the time at which the moving body left its position of rest and began to descend, the amount of speed acquired during the first two time-intervals will be double that acquired during the first time-interval alone; so the amount added during three of these time-intervals will be treble; and that in four, quadruple that of the first time-interval. To put the matter more clearly, if a body were to continue its motion with the same speed which it had acquired during the first time-interval and were to retain this same uniform speed, then its motion would be twice as slow as that which it would have if its velocity had been acquired during two time-intervals.

And thus, it seems, we shall not be far wrong if we put the increment of speed as proportional to the increment of time; [162] hence the definition of motion which we are about to discuss may be stated as follows: A motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed.

...[T]his is the customand properly soin those sciences where mathematical demonstrations are applied to natural phenomena, as is seen in the case of perspective, astronomy, mechanics, music, and others where the principles, once established by well-chosen experiments, become the foundations of the entire superstructure. I hope therefore it will not appear to be a waste of time if we discuss at considerable length this first and most fundamental question upon

which hinge numerous consequences of which we have in this book only a small number, placed there by the Author, who has done so much to open a pathway hitherto closed to minds of speculative turn. So far as experiments go they have not been neglected by the Author; and often, in his company, I have attempted in the following manner to assure myself that the acceleration actually experienced by falling bodies is that above described.

A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball. Having placed this[213] [179] board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse-beat. Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter the length of the channel; and having measured the time of its descent, we found it precisely one-halfof the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the plane, i. e., of the channel, along which we rolled the ball. We also observed that the times of descent, for various inclinations of the plane, bore to one another precisely that ratio which, as we shall see later, the Author had predicted and demonstrated for them.

For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times, and this with such accuracy that although the operation was repeated many, many times, there was no appreciable discrepancy in the results.