Use the test for your answers and work. If you need more space, write on the backs of the pages.

You may refer to books and notes, but you may not, in any form, seek assistance from, or offer assistance to, any person other than the instructor. Failure to abide by these instructions subjects you to penalties enumerated in the syllabus.

The test is worth 40 points. You may choose to complete EITHER 10 MULTIPLE CHOICE QUESTIONS (worth 1 point each) AND SIX (out of eight) PROBLEMS (worth 5 points each) OR EIGHT PROBLEMS (worth 5 points each).

There is one extra credit multiple choice question at the end worth one point.

____ Check here if you want the 10 multiple choice questions graded.

Circle or otherwise clearly indicate the right answer:

1. Consider what happens to the velocity and acceleration of a ball as it rolls down a hill like the one in the figure below.

   ![Diagram of a rolling object]

   (a) The magnitude of the velocity increases; the magnitude of the acceleration remains the same.
   (b) The magnitude of the velocity decreases; the magnitude of the acceleration remains the same.
   (c) The magnitude of the velocity increases; the magnitude of the acceleration decreases.
   (d) The magnitude of the velocity decreases; the magnitude of the acceleration increases.
   (e) The magnitude of both increases.
   (f) The magnitude of both decreases.
   (g) The magnitude of both is constant.
2. Two metal spheres are dropped one after the other (there is a time interval \( \delta t > 0 \) between the release of the two spheres). The first sphere dropped has a mass number \( m_1 \); the second one has a mass number \( m_2 \); \( m_2 > m_1 \). After the second sphere is released, the separation between them as they both fall

(a) increases with time.
(b) decreases with time.
(c) remains the same.
(d) depends on factors not mentioned in the problem, so one can’t tell.

3. The earth does not fall into the sun because

(a) there is no net force on it.
(b) the sun’s gravitational field is uniform.
(c) the gravitational attraction of the sun is very weak at this distance.
(d) the gravitational attraction of the moon, outer planets, and stars balances that of the sun and inner planets.
(e) all of the above.
(f) none of the above.

4. Consider the changes in momentum and mechanical energy of a bug and a car when the unfortunate bug splatters against the car’s windshield.

(a) The momenta and mechanical energies of bug and car change by the same amount.
(b) The momenta of bug and car change by the same amount, but the bug’s mechanical energy changes more than the car’s.
(c) The momenta of bug and car change by the same amount, but the car’s mechanical energy changes more than the bug’s.
(d) The mechanical energies of bug and car change by the same amount, but the car’s momentum changes more than the bug’s.
(e) The mechanical energies of bug and car change by the same amount, but the bug’s momentum changes more than the car’s.
(f) The momenta and mechanical energies remain the same.
(g) The momenta and mechanical energies change in proportion to the mass numbers.
(h) None of the above.
5. Three stones are thrown off a cliff into the ocean. The \textit{magnitude} of the initial velocity of each stone is the same, but the directions are different. The first stone (d) is thrown downward (vertically down); the second (h) is thrown straight out (horizontally), and the third stone (u) is thrown upward (vertically up). What is true of the relative velocities of the stones as they hit the water?

(a) $|\vec{v}_d| = |\vec{v}_h| = |\vec{v}_u|$.
(b) $|\vec{v}_d| > |\vec{v}_h| > |\vec{v}_u|$.
(c) $|\vec{v}_d| = |\vec{v}_h| > |\vec{v}_u|$.
(d) $|\vec{v}_d| > |\vec{v}_h| > |\vec{v}_u|$.
(e) $|\vec{v}_h| > |\vec{v}_u| > |\vec{v}_d|$.
(f) $|\vec{v}_h| = |\vec{v}_u| > |\vec{v}_d|$.
(g) $|\vec{v}_h| > |\vec{v}_u| = |\vec{v}_d|$.
(h) $|\vec{v}_u| > |\vec{v}_d| > |\vec{v}_h|$.
(i) $|\vec{v}_u| = |\vec{v}_d| > |\vec{v}_h|$.
(j) $|\vec{v}_u| > |\vec{v}_d| = |\vec{v}_h|$.
(k) None of the above.

6. When a plane banks or a car or train traverses a corner, we tend to feel as if we are being thrown to the outside of the vehicle. This sensation is attributed to a centrifugal force. The strength of this “force” would increase most if

(a) the radius of the turn were doubled.
(b) the magnitude of the instantaneous velocity during the turn were doubled.
(c) the radius of the turn were halved.
(d) either (a) or (b) occurred (because they produce the same effect).
(e) either (b) or (c) occurred (because they produce the same effect).
(f) something else happened because none of these make any difference.

7. If the momentum of an object in free fall doubles, its kinetic energy

(a) doubles.
(b) halves.
(c) quadruples.
(d) quarters.
(e) increases by some amount depending on the object’s mass number.
(f) remains the same.
(g) may or may not change, but I need more information to decide.
(h) changes, but none of the previous answers is right.
8. When a rocket fires from a launcher, the launcher recoils. Thus, instead of initially having zero momentum and zero kinetic energy, both rocket and launcher end up with non-zero momentum and kinetic energy. Which is necessarily true of the "after-launch" situation?

(a) \( \vec{p}_{\text{rocket}} = -\vec{p}_{\text{launcher}} \)
(b) \( \vec{p}_{\text{rocket}} = \vec{p}_{\text{launcher}} \)
(c) \( K_{\text{rocket}} = -K_{\text{launcher}} \)
(d) \( K_{\text{rocket}} = K_{\text{launcher}} \)
(e) Both (a) and (c).
(f) Both (a) and (d).
(g) Both (b) and (c).
(h) Both (b) and (d).
(i) None of the above.

9. An object, isolated in space and from all other objects and fields, can, without shedding any of its parts, change

(a) its own linear momentum.
(b) its own linear kinetic energy.
(c) its own angular momentum.
(d) its own rotational kinetic energy.
(e) (a) and (b).
(f) (c) and (d).
(g) (a) and (c).
(h) (b) and (d).
(i) (a) and (d).
(j) (b) and (c).
(k) (a), (b), and (c).
(l) (b), (c), and (d).
(m) (a), (b), and (d).
(n) (a), (c), and (d).
(o) (a), (b), (c), and (d).
(p) none of the above.

10. A simple pendulum consists of a ball attached to the end of string. When rocking back and forth, the ball’s velocity reaches maximum magnitude at the lowest point of its oscillation. The magnitude of the ball’s acceleration is a maximum

(a) at the lowest point, also.
(b) at the highest point.
(c) half way between lowest and highest.
(d) at some other position than any of these.
List the 6 problems you want graded: 

1. A block with mass number $m$ is projected with velocity $\vec{v}_i$ up a rough plane inclined at angle $\phi$. The coefficient of kinetic friction between the surfaces of the block and incline is $\mu_k$. The block slides up the incline, reverses direction, and slides down to the bottom. If the block takes twice as much time to come down as to go up the incline, (in terms of the information given and standard constants only)

   (a) draw free-body and force-vector diagrams for the block on the way up, at its highest position, and on the way down;

   (b) determine the magnitude of the block’s acceleration on its way up, at its highest position, and on its way down;

   (c) sketch $a$ vs $t$, $v$ vs $t$, and $s$ vs $t$ graphs for the block’s motion;

   (d) determine the relationship between $\mu_k$ and the angle $\phi$; and

   (e) calculate the fraction of the block’s initial kinetic energy lost to friction by the end of the round trip.
2. A rigid rod with mass number \( M \) and length \( \ell \) is clamped in a horizontal position to the end of a rotator shaft. Two very small objects with mass numbers \( m_1 \) and \( m_2 \), respectively, are attached each to an end of the rod [see figure below]. \( s \) is the distance from the position of \( m_1 \) to a position along the rod where the rotator shaft is clamped to the rod. Answer the following questions in terms of the information given and standard physical constants.

(a) If this system rotates with angular velocity \( \vec{\omega} \) around the center of the rod \( (s = \frac{\ell}{2}) \), what is its total moment of inertia?

(b) What should \( s \) be instead so as to minimize the work necessary to get the rod rotating in the horizontal plane from rest to an angular velocity \( \vec{\omega} \)? [Hint: Find \( s \) such that the rotational kinetic energy is a minimum.]

(c) What, if any, other physical significance besides meeting the energy requirement of (b) does this clamping position have?

(d) What constant torque must the clamp exert on the system if the system in this configuration reaches its final angular velocity after \( n \) full rotations?

(e) What must be the rotator’s minimum power rating under these conditions?
3. A solid homogeneous sphere with mass number \( M \) and radius \( \bar{r} \) starts with center of mass velocity \( \vec{v}_i \) down an plane inclined at angle \( \phi \). It rolls without slipping \( \Delta s \) (along the inclined surface) to the bottom of the incline.

(a) Draw free-body and force-vector diagrams for the sphere.

(b) What is the sphere’s angular velocity \( \vec{\omega} \) at the bottom of the incline?

(c) What is the magnitude of the sphere’s angular displacement \( \Delta \vec{\theta} \) as a result of its descent?

(d) What is the magnitude of the static friction in this case?

(e) Describe in words the energy transformations in this interaction. In particular, explain the effect of friction on the total mechanical energy.
4. A light string of length $\ell$ hangs from the ceiling. Attached to the end of the string is a small object with mass number $m$. Somehow (we won’t worry about the details), the object at the end of the string is made to rotate uniformly with angular velocity $\vec{\omega}$ in a horizontal circle while the string remains taut. In terms of the information given and standard physical constants,

(a) draw free-body and force-vector diagrams for the object when it is circling;
(b) determine the tension $\vec{T}$ in the string;
(c) determine $\theta$ the angle at which the string hangs while the object circles with angular velocity $\vec{\omega}$.
(d) describe (in words) the motion of the object if the string suddenly breaks;
(e) determine how far from the position of the string’s contact with the ceiling the object will land if the ceiling is height $h$ above the floor.
5. A child with mass number $m_c$ sits (B) on a platform holding one end of a light cord of length $\ell$ (see figure below). The cord is somehow secured at the other end (A); the child holds it horizontal. The child intends to drop off the platform holding on to the cord, swing down to a flotation device standing on a pier below (C), simultaneous release the cord and grab the flotation device, and fly off together into the lake height $h$ further below (D). The flotation device has mass number $m_f$.

(a) Sketch the child’s trajectory on the figure.
(b) In terms of the information given and standard physical constants, determine the child’s velocity $\vec{v}_b$ just before she releases the cord and grabs the flotation device, and the velocity $\vec{v}_a$ of child and flotation device immediately after releasing the cord and grabbing the device.
(c) Determine the child’s acceleration (magnitude and direction) just before she releases the cord and grabs the flotation device.
(d) Determine how far $s$ out from the pier the child and flotation device sail before splashing into the lake.
(e) Determine the magnitude and direction of the velocity of child and flotation device as they hit the water.
6. A light spring hangs from the ceiling. When a platform with mass number \( m_p \) is connected to the end, the spring stretches a distance \( \ell \) (see figure below). A ball with mass number \( m_b \) is released from height \( h \) above the platform, it bounces perfectly elastically off the platform and is caught before a second collision can occur. Complete the following problems in terms of the information given and standard physical constants.

(a) Determine \( k \) the spring constant.

(b) With what period \( T \), frequency \( f \), and “angular velocity” \( \omega \) will the platform oscillate?

(c) Determine \( S \) the amplitude or maximum displacement of the platform from its relaxed position?

(d) Sketch position vs clock reading, velocity versus clock reading, and acceleration versus clock reading graphs for the platform, assigning \( t_i = 0 \) the instant of the collision.

(e) Write an equation for the separation \( \Delta y \) as a function of the time interval \( \Delta t \) between the ball and the platform from the instant of the collision until the ball is caught.
7. A person with mass number \( m_p \) stands on the perimeter of a solid uniform circular platform with mass number \( m_d \) and radius of magnitude \( |\vec{\rho}| \). The platform can rotate smoothly on an axis through its center. Initially, both person and platform are at rest relative to the ground. The person starts walking and, after interval \( \Delta t \), reaches an instantaneous velocity relative to the ground of magnitude \( |\vec{v}| \). In terms of this information and standard physical constants,

(a) what is the person’s final angular velocity \( \vec{\omega}_p \) relative to an axis through the center of the platform and perpendicular to the plane on which the person stands?
(b) what is the platform’s final angular velocity \( \vec{\omega}_d \) relative to the same axis?
(c) what is the person’s angular velocity \( \vec{\omega} \) relative to the platform?
(d) how long does it take \( T \) the person to return to the same position on the platform when the person moves at maximum velocity relative to the ground?
(e) how much power has the person exerted in getting up to speed?
8. You will recall from class that for a damped (spring) harmonic oscillator, the critical damping \( b_{\text{crit}} = 2\sqrt{mk} \), where \( k \) is the spring constant and \( m \) is the mass number of the object at the end of the spring, results from the argument of the square root in the solution of the differential equation vanishing:

\[
\left( \frac{k}{2m} \right)^2 = |\vec{\omega}_0|^2, \quad \text{where} \quad |\vec{\omega}_0| = \sqrt{\frac{k}{m}} \quad \text{is the natural angular velocity of the simple harmonic oscillator.}
\]

Consider now a damped simple pendulum of length \( \ell \) with an object of mass number \( m \) at its free end. Remember in this case that the natural angular velocity is

\[
|\vec{\omega}_0| = \sqrt{\frac{|\vec{g}|}{\ell}}.
\]

(a) Draw free body and force vector diagrams for the object at the end of the simple pendulum.

(b) What is the value of \( b_{\text{crit}} \) for the simple pendulum in terms of the information given?

(c) What is the so-called “relaxation time” of the critically damped simple pendulum, that is, how long does it take the critically damped simple pendulum to reduce its amplitude to \( \frac{1}{e} \) of its original amplitude?

(d) How much energy is dissipated by the damping force(s) for the critically damped pendulum released with initial velocity \( \vec{v} \) (perpendicular to the line of the pendulum and in the plane of the non-dissipative forces acting on the object) from a small initial angular offset from vertical \( \theta_0 \)?

(e) Sketch \( \theta \) vs. \( t \), \( \omega \) vs. \( t \), and \( \alpha \) vs. \( t \) for the critically damped pendulum released with initial velocity \( \vec{v} \) (perpendicular to the line of the pendulum and in the plane of the non-dissipative forces acting on the object) from a small initial angular offset from vertical \( \theta_0 \).
Extra Credit: Two identical trucks lost their brakes going down the same hill under exactly the same weather and road conditions. The first truck was empty and traveling at velocity $\vec{v}$. The second truck was carrying a load whose collective mass number was the same as the truck’s (that is, the mass number of truck + load = 2× the mass number of the truck), but was traveling at velocity $\frac{1}{2}\vec{v}$. In order not to cause a serious accident, the drivers of both trucks directed their vehicles into an emergency stopping lane at the bottom of the hill. The emergency stopping lane was stacked with bales of hay. Which truck penetrated farther into the hay before coming to a stop?

(a) The first truck (empty but moving faster).
(b) The second truck (full–twice as heavy–and slow–moving half as fast–compared to the first).
(c) Each penetrated essentially the same distance.
(d) There is not enough information to tell.