Background Material: Exercise 1

1. The area of a right triangle of base, \( b \), and height, \( h \), is
   (a) \( bh \).
   (b) \( bh/2 \).
   (c) \( 2bh \).
   (d) \( \sqrt{b^2 + h^2} \).
   (e) none of the above.

2. The volume of a sphere of radius \( R \) is
   (a) \( \frac{4}{3} \pi R^3 \).
   (b) \( 4 \pi R^2 \).
   (c) \( \pi R^2 \).
   (d) \( 2 \pi R \).
   (e) none of the above.

3. The surface area of a sphere of radius \( R \) is
   (a) \( \frac{4}{3} \pi R^3 \).
   (b) \( 4 \pi R^2 \).
   (c) \( \pi R^2 \).
   (d) \( 2 \pi R \).
   (e) none of the above.

4. The area of a circle of radius \( R \) is
   (a) \( \frac{4}{3} \pi R^3 \).
   (b) \( 4 \pi R^2 \).
   (c) \( \pi R^2 \).
   (d) \( 2 \pi R \).
   (e) none of the above.

5. The circumference of a circle of radius \( R \) is
   (a) \( \frac{4}{3} \pi R^3 \).
   (b) \( 4 \pi R^2 \).
   (c) \( \pi R^2 \).
   (d) \( 2 \pi R \).
   (e) none of the above.
6. Velocity, \( \vec{v} \), has dimension \( LT^{-1} \). Time, \( t \), has dimension \( T \). Acceleration, 
\( \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \), has dimension 
(a) \( MLT^{-2} \).
(b) \( MLT^{-1} \).
(c) \( LT^{-2} \).
(d) \( ML^2T^{-2} \).
(e) none of the above.

7. Velocity, \( \vec{v} \), has dimension \( LT^{-1} \). Mass, \( m \), has dimension \( M \). Momentum, 
\( \vec{p} = m \vec{v} \), has dimension 
(a) \( MLT^{-2} \).
(b) \( MLT^{-1} \).
(c) \( LT^{-2} \).
(d) \( ML^2T^{-2} \).
(e) none of the above.

8. Kinetic energy, \( K \equiv \frac{1}{2}m |\vec{v}|^2 \), has dimension 
(a) \( MLT^{-2} \).
(b) \( MLT^{-1} \).
(c) \( LT^{-2} \).
(d) \( ML^2T^{-2} \).
(e) none of the above.

9. Force, \( \vec{F} = m\vec{a} \) has dimension 
(a) \( MLT^{-2} \).
(b) \( MLT^{-1} \).
(c) \( LT^{-2} \).
(d) \( ML^2T^{-2} \).
(e) none of the above.
10. One might guess that the frequency of a pendulum (essentially, how fast it rocks back and forth), might depend on the length of the pendulum, the mass of the pendulum, and the strength of the pull of gravity on the pendulum (as indicated by the gravitational acceleration). We can make a better guess by considering dimensions. The length, $\ell$, has dimension $L$; the mass, $m$, has dimension $M$; and the acceleration $\vec{a}$ has dimension $LT^{-2}$. Frequency, $f$, has dimension $T^{-1}$. Thus, we may write (recalling that brackets, $[\cdot]$, indicate “dimension of”)

$$[f] = [\ell]^a [m]^b [\vec{a}]^c \quad T^{-1} = L^a M^b (LT^{-2})^c,$$

where $a$, $b$, and $c$ are rational numbers that will make the dimensional equation balance.

In order for this to happen, $a$, $b$, and $c$ must be

(a) $a = 1 \quad b = 0 \quad c = -1$
(b) $a = -1 \quad b = 1 \quad c = 1$
(c) $a = 0 \quad b = 1/2 \quad c = -1/2$
(d) $a = -1/2 \quad b = 0 \quad c = 1/2$
(e) none of the above.

Problems 11-13 refer to the figure below:

11. The cosine of the angle, $\cos \theta =$

(a) $a/b$.
(b) $a/c$.
(c) $b/a$.
(d) $b/c$.
(e) none of the above.
12. The sine of the angle, \( \sin \theta = \)

(a) \( \frac{a}{b} \).
(b) \( \frac{a}{c} \).
(c) \( \frac{b}{a} \).
(d) \( \frac{b}{c} \).
(e) none of the above.

13. The tangent of the angle, \( \tan \theta = \)

(a) \( \frac{a}{b} \).
(b) \( \frac{a}{c} \).
(c) \( \frac{b}{a} \).
(d) \( \frac{b}{c} \).
(e) none of the above.