Determining $g$ with Mechanical Energy Conservation

If an interaction conserves total mechanical energy (that is, if the force is conservative),

$$KE + PE = \text{constant} \quad (1)$$

where KE stands for kinetic energy and PE stands for potential energy.

Where the only force involved is gravity, which is a conservative force, total mechanical energy is conserved, and the potential energy is the gravitational potential energy (near the surface of the earth),

$$PE = PE_G = mgh \quad (2)$$

where $m$ is the mass number of the object experiencing the gravitational force, $g$ is the acceleration due to gravity, and $h$ is the vertical position relative to some arbitrarily designated zero point. (Gravitational potential energy depends on position, and position is determined relative to an arbitrary origin: wherever one sets $h = 0$, $mgh = 0$. At, for example, a position below $h = 0$, $h < 0$, and therefore $mgh < 0$. All that matters in energy transformation calculations is change in potential energy; one cannot measure an amount of gravitational potential energy, only calculate the change from one position to another position by noting the change in some other form of energy—here, kinetic energy.)

If the object moves vertically, in one dimension, under the influence of gravity only, then, as the object goes up, kinetic energy decreases while potential energy increases such that the sum remains constant. The reverse happens as the object goes down. When the object passes through its highest position, the kinetic energy passes through a zero value while the potential energy reaches a maximum value, which we might write $PE_{\text{max}} = mgh_{\text{max}}$. Since the total mechanical energy is a constant value, it must equal $PE_{\text{max}}$, so, for this sort of system, Equation 1 may be rewritten

$$\frac{1}{2}mv^2 + mgh = mgh_{\text{max}} \quad (3)$$

Simplifying and rearranging,

$$v^2 = 2g(h_{\text{max}} - h) \quad (4)$$

which is an equation for a line of $v^2$ versus $(h_{\text{max}} - h)$, with slope $2g$. Thus, Equation 4 may be used to determine the acceleration due to gravity, $g$. You will use the configuration of Figure 1. Note that in this arrangement, the cart’s position along the track is not its vertical position, but rather the vertical position is the side opposite the hypotenuse of an imaginary right triangle formed.
from a line parallel to the table as the side adjacent and the cart’s displacement down the track as the hypotenuse. The cart is closest to the motion sensor when \( h = h_{\text{max}} \), because of the sensor’s placement at the top of the incline. Therefore, \( (h_{\text{max}} - h) < 0 \).

1. Determine and record the incline angle of the track.

2. Set up Data Studio with a motion sensor set to take data at 40 Hz. Set the motion sensor switch to record motion at short distances.

3. Press the Start button and launch the dynamics cart up the track so that it reverses direction before getting too close (15 cm) to the motion sensor. You may need a couple of practice trials. Adjust the launcher if the cart goes too high. Hold the track firmly so it will not recoil when the cart is launched.

4. Press the Stop button after the cart has bounced off the launcher.

5. On the position versus time graph, highlight data points around the point of the cart’s closest approach to the motion sensor. Take only points when the cart is free from the launcher. Copy the data and paste it into an Excel spreadsheet.

6. In a new column, calculate the cart’s vertical position at each instant (note that this will be down from the top of the incline).

7. Next to this column, calculate the difference between the maximum vertical position and each vertical position.

8. In another new column, calculate the cart’s velocity at each instant: Use the times and positions before and after the instant you are considering; for example, for the velocity at the second instant, subtract the position at the first instant from the position at the third instant and divide the result by the difference between the third and first times (there will be no velocities for the first or last instants).

9. Create a column of squared velocities.

10. Use the regression utility for squared velocities against vertical position differences to find the slope of the relationship. Calculate \( g \) and compare it to each of your previous determinations.