Conservation of Linear Momentum

In a closed system (no external forces), momentum is conserved: \( \sum \vec{p}_i = \sum \vec{p}_f \). Carts on a track are constrained to one-dimensional motion, so momentum conservation is expressed:

\[
m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \tag{1}
\]

Note that the velocities are signed quantities, not magnitudes: direction and sign must agree. The \( i \) indicates “initial” and the \( f \) indicates “final”, that is, before and after some sort of interaction.

The interactions you will be investigating are generally classified as collisions, of which there are, broadly speaking, three types: elastic, inelastic, and explosive. Most real-world collisions exhibit some amount of inelasticity.

In a perfectly elastic collision, kinetic energy is conserved:

\[
KE_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 = KE_f \tag{2}
\]

In an inelastic collision, kinetic energy is lost, consistent with momentum being conserved:

\[
KE_f < KE_i \tag{3}
\]

In an explosion separating a system at rest into two parts, the momenta of the two parts have equal magnitude but opposite directions:

\[
m_1 v_{1f} = -m_2 v_{2f}
\]

From this you should show that for a perfect explosion into two parts,

\[
\frac{KE_{1f}}{KE_{2f}} = \frac{m_2}{m_1} \tag{4}
\]

By the way, since, here, \( KE_{1i} = KE_{2i} = 0 \), \( KE_f > KE_i \) in an explosion.

You will investigate three collisions, checking whether, as expected, momentum is conserved in each case, and determining whether or not the kinetic energy description of each type of interaction holds: can the first interaction be classified as perfectly elastic (does \( KE_i = KE_f \)), is the second interaction inelastic (does \( KE_f < KE_i \)), is the last interaction explosive (does \( KE_1/KE_2 = m_2/m_1 \)). For each type of interaction you will perform one trial, in which \( m_1 \approx m_2 \). Use the digital scale for masses (uncertainty 2 in the last digit). For velocities, highlight 6-10 points just before and just after the collision and use the mean in Data Studio (you will also have to calculate the standard error of the mean by getting the standard deviation and the number of points from Data Studio) of the velocity versus time graph. You must propagate uncertainties for all further calculations (momentum, kinetic energy, etc.) in each case.

1. Attach the motion sensors on each end of the track to the interface both physically and virtually (i. e., in Data Studio). Set the range switches to short-range and the trigger rate to 50 Hz.

2. Orient two magnet-bearing carts so that the magnets face each other (keep watches and credit cards away from the magnets).

3. Start Data Studio and push one cart toward the other (which is stationary). The carts should not touch, but the motion should be rapid enough so that residual friction doesn’t swamp the interaction (the velocity versus time graph should be approximately horizontal just before and just after the collision). Stop Data Studio after the carts separate.

4. Find and record initial and final velocities for each cart, and perform the calculations to determine if this collision is consistent with being elastic.
5. Orient two carts so that their velcro pads face each other. Start Data Studio and push one cart at the other so that they collide and stick together, but keep rolling. Stop Data Studio.

6. Find and record initial and final velocities for each cart, and perform the calculations to determine if this collision is consistent with being inelastic.

7. Place two carts in contact, with the plunger of one completely retracted and latched. Start Data Studio and gently tap the plunger-release button. Stop Data Studio.

8. Find and record initial and final velocities for each cart, and perform the calculations to determine if this collision is consistent with being explosive.