Determining $g$ with Newton’s 2nd Law and Friction

According to Newton’s 2nd Law

$$\sum_i \vec{F}_i = m\vec{a}, \quad (1)$$

the vector sum of the forces acting on an object with mass number $m$ results in an acceleration $\vec{a}$ such that the product $m\vec{a}$ is numerically equal to the vector sum of forces $\sum_i \vec{F}_i$, and the direction of the acceleration is the same as that of the resultant force. If the vector sum of forces equals zero, then the magnitude of the acceleration will be zero, and the object will continue to move in the same direction at constant velocity (if the velocity was zero, it will remain zero).

Friction is a contact force that opposes the motion. When the friction is between two surfaces, two sorts of friction may be distinguished, depending on the relative motion of the surfaces. In both cases, the magnitude of the resistive (frictional) force is related to the magnitude of the normal force of one surface on the other, $N$. If the surfaces are at rest relative to one another, the contact friction is referred to as static friction. It’s magnitude varies, under the influence of an external force parallel to the surfaces, from zero (if the external force is zero) to a maximum value at which the surfaces begin to slide relative to one another. That maximum value depends not only on the normal force, but also on the characteristics of the surfaces (what materials they are made of, as well as their smoothness, dryness, and cleanliness). These characteristics are subsumed in a dimensionless number called the coefficient of static friction, $\mu_s$. Then the magnitude of the static friction force is given by

$$f_s \leq \mu_s N. \quad (2)$$

When the surfaces are moving relative to one another, the resistive (frictional) force is known as kinetic friction, which similarly depends on the normal force and, in a different way, on surface characteristics. The dimensionless number that subsumes these characteristics is designated $\mu_k$, and the magnitude of the kinetic friction force is given by

$$f_k = \mu_k N. \quad (3)$$

In general $\mu_k < \mu_s$, which is why it typically takes more effort to start an object sliding than to keep it sliding. To keep the object sliding at constant velocity, the component of the external force parallel to the surface must equal $f_k$. If constant velocity sliding occurs on an inclined plane [see Figure 1], then

\[
\begin{align*}
N &= mg \cos \theta \\
f_k &= mg \sin \theta
\end{align*}
\]

Figure 1: Forces affecting an object on an inclined plane.

Using Equations 3, 4, and 5, you should be able to show that

$$\mu_k = \tan \theta \quad (6)$$

For constant velocity on a flat surface [see Figure 2],
Figure 2: Forces affecting an object on a horizontal surface.

\[ f_k = F \]  \hspace{1cm} \text{(7)}
\[ N = W \]  \hspace{1cm} \text{(8)}

Since \( W = mg \), you should be able to show, by combining Equations 3, 6, 7, and 8, that

\[ g = \frac{F}{m \tan \theta} \]  \hspace{1cm} \text{(9)}

Finding \( g \) through Equation 9 is the objective of the following investigation.

1. Determine the mass of the friction cart with the cork bottom. Add 500 g to the cart. Use a mass uncertainty of 2 g.
2. Set up Data Studio to use the motion sensor. You’ll be looking for constant velocity on the velocity versus time graph.
3. Adjusting the height of one end of the track, try to find the angle at which the cart, once you get it moving, slides at constant velocity. Measure the angle. Assume an uncertainty of 3°.
4. Set up Data Studio to use the force sensor.
5. With the track horizontal, attach the force sensor to the cart.
6. Start Data Studio and tare (zero) the force with no tension in the connecting string.
7. Find the force (taking the average by selecting a portion of the force versus time graph and selecting mean from the \( \Sigma \) menu) which results in the cart moving at constant velocity (as shown in the velocity versus time graph). Be sure to pull in such a way that the connecting string remains horizontal. Repeat so as to get four good force measurements. Average these, and find the standard deviation and standard error of the mean to get a value for the force which moves the cart at constant velocity along a horizontal track.
8. Calculate \( g \) with Equation 9, propagate uncertainties, and compare this result with previous determinations.