Determining \( g \) with a Physical Pendulum

A simple pendulum consists of a small weight with mass number \( m \) at the end of a light string of length \( \ell \). The torque on the weight due to the force of gravity when the string is displaced an angle \( \theta \) from the vertical is

\[
\tau = m g \ell \sin \theta
\]

The moment of inertia of such a weight is \( m \ell^2 \), and so we get

\[
\tau = I \alpha
\]

\[
mg \ell \sin \theta = m \ell^2 \alpha
\]

For small angles \( \sin \theta \approx \theta \). Also, recall that \( \alpha \equiv \ddot{\theta} \). Simplifying and rearranging Equation 2,

\[
\ddot{\theta} = \frac{g}{\ell} \theta
\]

a second-order, linear differential equation whose solution is periodic (sinusoidal), with an angular velocity \( \sqrt{g/\ell} \), and therefore, the period of a simple pendulum is

\[
T = 2\pi \sqrt{\frac{\ell}{g}}
\]

A physical pendulum is an extended body with mass number \( M \) and moment of inertia \( I \), which rocks on a pivot displaced from the body’s center of mass a distance \( H \). The torque due to the force of gravity on the body’s center of mass is

\[
\tau = MgH \sin \theta
\]

and the equivalent equation of motion to Equation 3 is

\[
\ddot{\theta} = \frac{MgH}{I} \theta
\]

and the period of small oscillation, then, should be

\[
T = 2\pi \sqrt{\frac{I}{MgH}}
\]

The experiment you will be doing uses a rod of mass number \( m_{rod} \) with a small weight \( m \) attached to its lower end. If the rod has length \( \ell \), then its moment of inertia with respect to its center of mass (the middle of the rod) is

\[
I_{rod, \ cm} = \frac{1}{12} m_{rod} \ell^2
\]

If the point of rotation is displaced from the center of mass by a distance \( h \), then, by the parallel axis theorem, the rod’s moment of inertia is increased by \( m_{rod} h^2 \). The weight adds its moment of inertia, so the total moment of inertia (using \( d \) as the distance of the weight from the pivot point) is

\[
I = m_{rod} \left( \frac{\ell^2}{12} + h^2 \right) + md^2
\]

Thus, for this particular extended object, the period of small oscillation is

\[
T = 2\pi \sqrt{\frac{m_{rod} \left( \frac{\ell^2}{12} + h^2 \right) + md^2}{g(m_{rod}h + md)}}
\]
The acceleration due to gravity, \( g \), can then be determined by measuring the physical characteristics of this physical pendulum and the period of its oscillation at small amplitudes (angles of displacement).

\[
g = \frac{4\pi^2 \left( \frac{\ell^2}{12} + h^2 \right) + md^2}{T^2(m_{\text{rod}}h + md)} \tag{11}
\]

1. Find the mass numbers of the rod and the weight, \( m_{\text{rod}} \) and \( m \). Assume an uncertainty of 1 g for each. Measure the relevant lengths, and estimate their uncertainties.

2. Set up the pendulum to oscillate between the U-sides of a photogate sensor. The photogate sensor will measure the period of oscillation as it passes through the U-sides of the sensor.

3. Configure Data Studio to use the photogate sensor. A full period is the time from the pendulum’s first pass through the photogate until it passes again in the same direction. Therefore, three passes constitute one period. Press the Timer button on the Experiment Setup window, and a Timer Setup window will appear. Choose 'Blocked' three times on the photogate pull-down menu. Rename the timing sequence 'Period' and then choose Done. In the Data section a timer with the word 'Period' will appear.

4. Configure Data Studio to use the rotational motion sensor. Because the theory predicts the motion for small oscillation, we are interested in the angular displacement of the pendulum. Click on Measurements and make sure only Angular Position is selected. In the Rotational Motion Sensor tab, select Other under the Linear Calibration tab and click OK.

5. Press the Start button before starting to motion, so the rotational motion sensor can zero. Then gently push the pendulum to get it rocking with amplitudes of less than 10°. Put angular position and Period on the same graph.

6. Looking at the data after it has settled down with an oscillation amplitude smaller than 10°, highlight a section of the period versus time graph. From the \( \Sigma \)-icon, select mean, standard deviation, and count. From the standard deviation and count, calculate the standard error of the mean. The mean and its error comprise the result for the period.

7. Using Equation 11, calculate the acceleration due to gravity. In a step-by-step manner, propagate uncertainties through the equation. Compare your result with previous determinations.