Determining $g$ with Rotational Motion

The equation of rotational motion is analogous to that of linear motion, in which the standard form of Newton’s 2nd Law is $\sum F = ma$:

$$\sum \tau = I \alpha \quad (1)$$

where $\tau$ is torque, $I$ is moment of inertia, and $\alpha$ is angular acceleration. Just as the mass number indicates the degree of difficulty of linearly accelerating an object (inertia), the moment of inertia indicates the degree of difficulty of angularly accelerating an extended object. The magnitude of an (extended) object’s moment of inertia depends on both the magnitude of its mass number as well as its shape. A disk, with mass number $M$ and radius $R$, for example, has moment of inertia

$$I_{\text{disk}} = \frac{1}{2} MR^2 \quad (2)$$

The rotary motion sensor you will use in this investigation has a fly wheel which is approximately a disk with mass number $m_{\text{fw}}$ and radius $r_{\text{fw}}$. It is set into motion by the tension $T$ in a string wrapped around its perimeter. At the other end of the string is a weight of mass $m$ that is pulled by gravity. This pull, resisted by the inertia of the fly wheel, creates the string’s tension. The hanging mass, then, will accelerate downward with magnitude $a$ according to Newton’s 2nd Law:

$$mg - T = ma$$

or, solved for the tension:

$$T = m(g - a) \quad (3)$$

The string’s point of contact on the fly wheel will also have an instantaneous linear acceleration of $a$, and therefore an angular acceleration

$$\alpha = \frac{a}{r_{\text{fw}}}$$

Rearranging, we can substitute $a = r_{\text{fw}} \alpha$ into Equation 3:

$$T = m(g - r_{\text{fw}} \alpha) \quad (4)$$

The tension acts at a point displaced from the fly wheel’s center of mass, and so exerts a torque, accelerating the fly wheel angularly. The tension is perpendicular to the line connecting the point at which it acts to the axis of rotation, at a distance $r_{\text{fw}}$, so

$$\tau = r_{\text{fw}} T$$

Multiplying both sides of Equation 4, then, by $r_{\text{fw}}$, gives the equation for the torque on the system:

$$\tau = mr_{\text{fw}}(g - r_{\text{fw}} \alpha) \quad (5)$$

But we know from Equation 1 that $\tau = I \alpha$. In the system you will be investigating, the total moment of inertia is the sum of the moments of the fly wheel and the disk (see Equation 2):

$$I = I_{\text{fw}} + I_{\text{disk}} = \frac{1}{2} m_{\text{fw}} r_{\text{fw}}^2 + \frac{1}{2} m_{\text{disk}} r_{\text{disk}}^2 \quad (6)$$

Combining Equations 5 and 6, and rearranging, gives the acceleration due to gravity in terms of this accelerating rotational system:

$$g = \left( \frac{m_{\text{fw}} r_{\text{fw}}^2 + m_{\text{disk}} r_{\text{disk}}^2}{2mr_{\text{fw}}} + r_{\text{fw}} \right) \alpha \quad (7)$$

where, again, $m$ (no subscript) is the mass number of the weight at the end of the string.

You will use Equation 7 to determine the acceleration due to gravity from data collected in the following experiment.
1. Measure the mass numbers of the fly wheel, (50 gm) hanging weight, and disk. Determine the radii of fly wheel and disk.

2. Connect the rotary motion sensor to the interface and the disk to the rotary motion sensor. Wind the string around the fly wheel such that when the hanging weight is released the angular position will give positive values (the sensor should turn counterclockwise looking down).

3. Set the sensor acquisition rate to 20 Hz, and select all kinematic quantities to use radians.

4. Start data acquisition and release the weight. Stop data acquisition before the weight hits the floor.

5. Determine the constant angular acceleration by finding the slope of the angular velocity versus time graph.

6. Determine $g$ with Equation 7, propagating uncertainties, and compare this to all previous determinations.