Determining \( g \) with the Work-Kinetic Energy Theorem

When the net force acting on an object with mass number \( m \) has magnitude \( F \), the result, by Newton’s 2nd Law, is an acceleration of magnitude \( a \) in the direction of \( F \):

\[
F = ma
\]

which can be rearranged so the acceleration is the dependent variable:

\[
a = \frac{F}{m}
\]

If \( F \) is constant, then so is \( a \). When this is the case, the kinematic equations apply. Recall the kinematic equation in one dimension relating displacement (\( \Delta s \)), velocity (\( v \)), and acceleration, but not time:

\[
v_f^2 - v_i^2 = 2a\Delta s
\]

This tells us that if we plot the change in an object’s velocity squared against that object’s displacement, we should get a straight line with the slope being twice the acceleration. Substituting Equation 2 in Equation 3,

\[
v_f^2 - v_i^2 = 2 \left( \frac{F}{m} \right) \Delta s
\]

Rearranging,

\[
\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = F\Delta s
\]

Each term on the left side of Equation 4 is called kinetic energy, while the term on the right side is called work. Strictly speaking, work is a scalar (dot) product of two vectors: \( W = \vec{F} \cdot \Delta \vec{r} \), but here, in one dimension, the displacement (\( \Delta s \) in one dimension) is (anti-)parallel to the force, so we can write it \( F\Delta s \).

In the experiment you will perform, a “frictionless” cart with mass number \( m \) will be accelerated due to tension in a string pulled by a weight at its opposite end. If the mass number of the weight is \( M \), and knowing that the accelerations of both cart and weight have the same magnitude \( a \), you should be able to use Newton’s Second Law on both cart and weight to show that, the tension can be written, in terms of the acceleration due to gravity,

\[
T = \frac{mM}{m+M}g
\]

Since the force in Equation 4 \( F = T \), you should be able to show:

\[
v_f^2 - v_i^2 = \frac{2M}{m+M}g\Delta s
\]

So, if the change of the cart’s squared velocity is plotted against its displacement, the result should be a straight line with the slope being \( \frac{2M}{m+M} \) times the acceleration due to gravity.

You will complete the following experiment, depicted in Figure 1, and determine the acceleration due to gravity with the help of Equation 7. You will compare this determination to each of your other ones.

1. Configure the Data Studio interface with a motion sensor set to take data at 50 Hz. Set the switch at the top of the motion sensor to record motion at short distances.

2. Connect a piece of string to the cart and run the other end of the string over the pulley (attached to the end of the track opposite the motion sensor). Attach a 100g mass to the free end of the string and leave it hanging over the pulley.
3. With the cart at least 15 cm away from the motion sensor, start the motion sensor and release the cart, holding the track firmly so it will not recoil. Stop the cart before it hits the end of the track.

4. Highlight the cart’s position versus time data after it’s release and before stopping it. Copy the data to Excel. Compute all quantities necessary to find the slope of the line given by Equation 7: $\Delta v^2$ and $\Delta s$.

5. Use the regression utility to find the slope and the uncertainty of the slope.

6. Find $g$ by operating on the slope (and uncertainty) with the proper factor.

7. Compare this $g$ with those you have found in other investigations.