Basic Analog Circuits

Resistance and Ohm's Law

Current: \( I[A] = \frac{\Delta Q[C]}{\Delta t[s]} \); \( [A] \equiv [C][s]^{-1} \)
Potential Difference: \( V[V] = \frac{\Delta E[J]}{\Delta Q[C]} \); \( [V] \equiv [J][C]^{-1} \)
Resistance: \( R[\Omega] \)

Ohm's Law:

\[
V = IR
\]

To measure **current** through a component, connect an ammeter **in series** with it; to measure **potential difference** across a component, connect a voltmeter **in parallel** with it.
Power: \( P[W] = \Delta E[J] / \Delta t[s]; [W] \equiv [J][s]^{-1} \)
\[
P = IV = I^2R = \frac{V^2}{R}
\]

Note: Power dissipation limits the voltage that can be applied to the input of a digitizer. The square law indicates that doubling the input voltage requires the digitizer to dissipate four times more power.

**Voltage Divider**

\[
V_{out} = V_{in} \frac{R_2}{R_1 + R_2}; \quad R_{eq} = R_1 + R_2
\]

**Current Divider**

\[
I_1 = \frac{V}{R_1}; \quad I_2 = \frac{V}{R_2}; \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}
\]

**Kirchhoff’s Rules**

- Loop Rule: \( \Sigma (\text{voltage drops around a closed loop}) = \Sigma (\text{voltage sources}) \)
- Node Rule: \( \Sigma (\text{current into a node}) = \Sigma (\text{current out of a node}) \)

**Capacitance**

The electric field between the plates of a capacitor resists changes in applied voltage. Capacitors decrease their resistance with frequency.

Capacitance: \( C[F] = Q[C] / V[V]; [F] \equiv [C][V]^{-1} \)
Capacitance Circuit

Series:

\[ C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \]

Capacitors in Series

Parallel:

\[ C_{eq} = C_1 + C_2 \]

Capacitors in Parallel
**Inductance**

Inductance is the voltage drop across a component at a given rate of change of current through it. Mutual inductance is the voltage drop across a component at a given rate of change of current through a neighboring component. Inductors increase resistance with frequency.

Inductance: \( L[H] = \frac{V[V]}{dI[A]/dt[s]} \)

Mutual Inductance: \( M[H] = k\sqrt{L_1[H]L_2[H]} \) (\( k \) is a coupling coefficient dependent on the orientation of the inductors, \( 0 \leq k \leq 1 \))

![Inductor Circuit](image1)

**Series:**

\[ L_{eq} = L_1 + L_2 \pm 2M \]

![Inductors in Series](image2)

**Parallel:**

\[ L_{eq} = \frac{L_1L_2 \pm M^2}{L_1 + L_2 \pm 2M} \]

![Inductors in Parallel](image3)

**Alternating Current**

Periodically varying voltage and current are referred to as alternating (as opposed to direct): AC or DC. Alternating voltage may be described with a complex form:

\[ V(w, t) = V_0 e^{i\omega t} \]

where \( V_0 \) is the amplitude (magnitude of the maximum or minimum voltage) and \( \omega \) is the angular
velocity, related to the frequency by

\[ \omega = 2\pi f \]

**Impedance and Admittance**

Impedance is a measure of a circuit's hindrance to current from an applied alternating voltage. It is comprised of resistance and reactance. Reactance is the hindrance from capacitance and inductance. All are frequency dependent and are measured in units or resistance, \( \Omega \). Admittance is the inverse of the impedance.

Capacitive Reactance: \( X_C(\omega) = \frac{1}{\omega C} \)
Inductive Reactance: \( X_L(\omega) = \omega L \)
Reactance: \( X(\omega) = X_L(\omega) - X_C(\omega) = \omega L - \frac{1}{\omega C} \)
Impedance: \( Z(\omega) = \frac{V(\omega)}{I(\omega)} = R + iX(\omega) = R + i\left[X_L(\omega) - X_C(\omega)\right] = R + i\omega L + \frac{1}{i\omega C} \)
Admittance: \( A(\omega) = \frac{1}{Z(\omega)} \)

**Filters**

Electronic filters remove or select specific frequency ranges or components from signals. The simplest ones employ pairs of the passive components—resistors, capacitors, and inductors—we've already discussed. Recall that the impedances of capacitors and inductors (but not of resistors, which are not reactive elements) are frequency dependent: \( X_C = \frac{1}{\omega C}, X_L = \omega L \). Notice that capacitive reactance is large at low-frequency and small at high frequency, while inductive capacitance is the reverse. Thus, a high-frequency signal passing through a capacitor will be attenuated less than a low-frequency signal (high-pass filter). The reverse is true for an inductor. While resistors are not reactive, in combination with a reactive element, they determine the time-constant of the circuit, and therefore its frequency response range.

![High-Pass RC Filter](image)

![Low-Pass RL Filter](image)

Notice that in both of these circuits, the resistor is parallel to the power source, with one end tied to ground. Reversing the positions of components, reverses the behavior (remember, that impedance behaves like resistance, adding directly in series but inversely in parallel).
A band-pass filter, in which an intermediate frequency range is able to pass, can be created by combining a low-pass filter with a high-pass filter, or, equivalently, with an RLC circuit.

**Diodes**

Diodes are semiconductor devices which pass current in one direction.

Typical diode current vs. voltage diagram. Conduction of the forward current commences at between 0.2 and 0.7 volts, depending on the kind of diode.

Extremely versatile passive components, diodes can regulate voltages; tune, frequency-multiply, and
mix radio-frequency circuits; emit light; switch; and perform logic in digital circuits.

Simple diode-resistor rectifier.

**Analog Amplifier Circuits**

Amplifiers increase the voltage or amplitude of signals.

\[ v_i(t) \rightarrow g(\cdot) \rightarrow v_o(t) \]

Input voltage increased by factor \( g \), the gain, to yield output voltage

**Differential amplifiers** amplify the difference between two voltages.

**Operational amplifiers** (op-amps) employ differential amplifiers with very high input impedance and large gain. The high input impedance implies very little input current, so signals are not distorted, and the large gain implies a capability of amplifying small signals.
In the limit of very small signals, the two input voltage levels are approximately equal, $V_{\text{in}}^- = V_{\text{in}}^+$. High input resistance implies $I_{\text{in}}^- = I_{\text{in}}^+ = 0$.

Referring to the basic operational amplifier depicted above, we note that resistors $R1$ and $R2$ form a voltage divider. If we designate the point at which $R1$ and $R2$ join as $V$, then we know that the voltage drop between $V$ and $V_{\text{out}}$ is $V - V_{\text{out}} = (V_{\text{in}} - V_{\text{out}}) \frac{R_2}{R_1 + R_2}$. But, here, by our assumptions, $V = V_{\text{in}}^- = V_{\text{in}}^+$, and $V_{\text{in}}^+$ is at ground, so $V = 0$. Substituting, and solving for $V_{\text{out}}$, we find

$$V_{\text{out}} = -V_{\text{in}} \frac{R_2}{R_1}.$$  

The minus sign indicates that the signal is inverted. The ratio $\frac{R_2}{R_1}$ is the gain factor, which can, in principle, range from infinitesimally small to infinitely large. Ratios greater than 1 indicate amplification; ratios less than 1 indicate attenuation.

Non-inverting op-amps always amplify.

\[ \text{Non-Inverting Operational Amplifier} \]

In this configuration, the voltage divider is between $V_{\text{out}}$ and ground, so

$$V = V_{\text{in}}^- = V_{\text{in}}^+ = V_{\text{in}} = V_{\text{out}} \frac{R_1}{R_1 + R_2}.$$  

Therefore,

$$V_{\text{out}} = V_{\text{in}} (1 + \frac{R_2}{R_1}).$$

Op-amps are also used for, among other things, buffering or isolating, comparing, summing, integrating, and differentiating signals.

Signal buffering or isolating is accomplished with a voltage follower circuit. This name comes from the fact that in this circuit, the output voltage is the same as the input voltage. There is no amplification or attenuation; the gain is 1. As we know, the op-amp has very large input impedance, so very little current—and, hence, very little power—is drawn from the input circuit. The input signal, therefore, is little affected, and the output signal is the same as the input. The result is an isolation buffer: the signal is transmitted with almost no power drawn from the circuit.
**Voltage Follower**

\[ V_{out} = V_{in}^- = V_{in}^+ = V_{in} \]

The extremely large (unloaded, or open-loop) inherent gain of an op-amp (as much as a factor of 10,000) can be exploited to indicate in a simple binary fashion when certain conditions are met at the inputs. One such application is the comparator circuit, in which the output indicates that a voltage from a source (such as a sensor of some kind) is higher or lower than a preset reference value.

\[ V_{REF} \]

In the diagram, the reference voltage is at the non-inverting input, while the signal comes in at the inverting input. When the magnitude of the signal exceeds that of the reference voltage, the output goes to its maximum value (in practice, that of the power lines into the op-amp, here \( \pm V_s \)), and stays there as long as the signal value exceeds the reference value. In this example, because the signal comes in the inverting input, the output is inverted. This is not a robust comparator—its transition is slow, so not up to handling high frequency signal changes, and it is subject to noise fluctuations—but the idea is illustrated.

A summing amplifier—or voltage adder—outputs a scaled sum of input voltages.
Op-Amp Summing Amplifier

Again, because $I_{in}^- = 0$, $I_1 + I_2 = I_F$, and because $V_{in}^- = V_{in}^+ = 0$, $I_1 = \frac{V_1}{R_1}$, $I_2 = \frac{V_2}{R_2}$, and $I_F = -\frac{V_{out}}{R_F}$. Therefore $V_{out} = -R_F \left( \frac{V_1}{R_1} + \frac{V_2}{R_2} \right)$. The scaled sum extends likewise with each additional input.

Recall that the capacitance, $C = Q/V$, and the current, $I = \Delta Q/\Delta t$. Therefore, $I = C \Delta V/\Delta t$. We will use this fact to demonstrate how an op-amp can integrate and differentiate signals.

![Op-Amp Integrator](image)

Op-Amp Integrator

Because $V_{in}^- = V_{in}^+ = 0$, and $I_{in}^- = 0$, $I_R = I_C = \frac{V_{in}}{R} = \frac{dV_C}{dt} = -C \frac{dV_{out}}{dt}$. Thus, $V_{out} = -\frac{1}{RC} \int V_{in} dt$.

![Input](image)

Input

![Output](image)

Output

Output from Square-Wave Input to an Op-Amp Integrator
Op-Amp Differentiator

By reasoning similar to the integrator, \( I_R = I_C = C \frac{dV_{in}}{dt} = -\frac{V_{out}}{R} \). Therefore, \( V_{out} = -RC \frac{dV_{in}}{dt} \).

Examples of Differentiator Behavior