HW Assignment 2: Due Feb. 17th

1)  
   a) Beginning with Kepler’s third law (Eq 2.35 in your reading in C&O ch. 22), derive an expression for $\Theta(R)$, assuming that the Sun travels in a Keplerian orbit about the center of the Galaxy.
   b) From your results in part (a), derive analytical expressions for the Oort constants A and B.
   c) Determine the numerical values for A and B in the solar neighborhood, assuming $R_0 = 8.0$ kpc and $\Theta_0 = 220$ km/s. Express your answer in units of km s$^{-1}$ kpc$^{-1}$.
   d) Do your answers in part c) agree with the measured values with the Milky Way Galaxy? Why or why not?
   (This is problem 22.17 in C&O)

2)  
   a) Estimate $d\Theta/dR$ in the solar neighborhood assuming that the Oort constants A and B are +14.4 and −12.0 km s$^{-1}$ kpc$^{-1}$, respectively. What does this say about the variation of the rotational velocity as a function of R near the Sun?
   b) If A and B were +13 and −13 km km s$^{-1}$ kpc$^{-1}$, respectively, what would the value of $d\Theta/dR$ be? What would this say about the shape of rotation curve in the solar neighborhood?

3)  
   a) Show that rigid body rotation near the Galactic center is consistent with a spherically symmetric mass distribution of constant density.
   b) Is the distribution of mass in the dark matter halo (Eq. 22.38 in C&O) consistent with rigid body rotation near the Galactic center? Why or why not?

4)  
   a) Assuming that Eq. 22.38 in C&O is valid for any arbitrary distance from the center of the Galaxy, show that the amount of dark matter interior to radius r is given by the expression:

   \[ M(R) = 4\pi C_0 \left[ r - a \tan^{-1} \left( \frac{r}{a} \right) \right] \]

   b) If $5.5 \times 10^{11}$ M$\odot$ of dark matter is located within 100 kpc of the Galactic center, determine $C_0$ in units of M$\odot$ kpc$^{-1}$. Repeat your calculation if $1.3 \times 10^{12}$ M$\odot$ is located within 230 kpc of the Galactic center.
5) In this problem, you will construct a crude model for the mass distribution and velocity curve in the inner 1 kpc of the Galaxy. Assume that a point (a black hole) of mass $M\odot = 4 \times 10^6 \, M\odot$ is located at the center of the Galaxy and that the remainder of the mass has an isothermal density distribution that varies as $r^{-2}$.

a) If the mass distribution is spherically symmetric, show that the mass interior to a radius $r$ can be expressed as a function of the form:

$$ M_r = kr + M_\odot $$

where $k$ is a constant to be determined.

b) Assuming perfectly circular motion and Newtonian gravity, show that the orbital velocity curve is given by:

$$ v = \left[ G\left( k + \frac{M_\odot}{r} \right) \right]^{1/2} $$

c) If the orbital velocity is 110 km/s at 2 pc, determine the value for $k$.

d) Plot $\log M_r$ as a function of $\log r$ over the range $0.01 \, \text{pc} < r < 1 \, \text{kpc}$. Express $M_r$ in solar units and $r$ in parsecs.

e) Plot $v$ as a function of $\log r$ over the range $0.01 \, \text{pc} < r < 1 \, \text{kpc}$. Express $v$ in units of km/s and $r$ in parsecs. At what radius does the contribution of the central point mass begin to become significant?