Final, Solutions

1. (a) 
(b) Shape unchanged but shifted up or down depending on magnitude and sign of initial velocity.

2. (a) \( -|v_0y| \), from \( v_y^2 = v_{0y}^2 + 2a\Delta y \)
(b) Equal, same velocity at same \( y \), same acceleration.
(c) Ball \( B \) will be greater. \( A \) loses mechanical energy on the way up and down, so, at release point, the two balls will not have the same velocity, as when there is no air resistance.
(d) Ball \( C \) greater. \( y \)-components the same, but \( C \) also has non-zero \( x \)-component.

3. The relevant equations come from:

\[
\begin{align*}
v_x &= v_{0x} = v_0 \cos \alpha \\
x &= v_{0x}t = v_0 \cos \alpha t \\
v_y &= v_{0y} - gt = v_0 \sin \alpha - gt \\
    &= v_{0y} - \frac{gx}{v_{0x}} = v_0 \sin \alpha - \frac{gx}{v_0 \cos \alpha} \quad (1) \\
y &= v_{0y}t - \frac{1}{2}gt^2 = v_0 \sin \alpha t - \frac{1}{2}gt^2 \\
    &= v_{0y} \frac{x}{v_{0x}} - \frac{gx^2}{2v_{0x}^2} = x \tan \alpha - \frac{g}{2v_0^2 \cos^2 \alpha} x^2 \quad (2)
\end{align*}
\]

For \( x = L \), we get from (1) and (2)

\[
\begin{align*}
v_y &= v_0 \sin \alpha - \frac{gL}{v_0 \cos \alpha} \quad (3) \\
y &= L \tan \alpha - \frac{g}{2v_0^2 \cos^2 \alpha} L^2 \quad (4)
\end{align*}
\]
• At $y = 0$ we can define a $v_{0, crit}$:

\[
0 = L \tan \alpha - \frac{g}{2v_{0}^2 \cos^2 \alpha} L^2
\]

\[
L \tan \alpha = \frac{g}{2v_{0}^2 \cos^2 \alpha} L^2
\]

\[
v_{0}^2 = \frac{gL}{2 \sin \alpha \cos \alpha}
\]

\[
v_{0, crit} = \sqrt{\frac{gL}{\sin 2\alpha}}
\]

• The projectile is still rising if $v_y > 0$ when $x = L$. So, from (3)

\[
v_{0} \sin \alpha - \frac{gL}{v_{0} \cos \alpha} > 0
\]

\[
v_{0} > \sqrt{2v_{0, crit}}
\]

• The objects meet below the initial level if $y < 0$ when $x = L$. So, from (4)

\[
L \tan \alpha - \frac{g}{2v_{0}^2 \cos^2 \alpha} L^2 < 0
\]

\[
L \tan \alpha < \frac{g}{2v_{0}^2 \cos^2 \alpha} L^2
\]

\[
v_{0}^2 < \frac{gL}{\sin \alpha \cos \alpha}
\]

• The projectile reaches the initial level before reaching the $x$-position of the target if $y = 0$ when $x < L$. So, from (2)

\[
0 = x \tan \alpha - \frac{g}{2v_{0}^2 \cos^2 \alpha} x^2
\]

\[
x \tan \alpha = \frac{g}{2v_{0}^2 \cos^2 \alpha} x^2
\]

\[
\frac{2v_{0}^2 \sin \alpha \cos \alpha}{g} = x
\]

\[
\frac{v_{0}^2 \sin 2\alpha}{g} < L
\]

\[
v_{0} < \sqrt{\frac{gL}{\sin 2\alpha}}
\]

\[
v_{0} < v_{0, crit}
\]
4. (a) 
(b) see figure 
(c) A parabola is a quadratic equation, second order and linear. It is the locus of points equidistant from a point (the focus) and a line (the directrix). The equation relates horizontal and vertical positions in space. Given the initial conditions of a motion, the equation predicts where the object will be given one of the coordinates: in the case of projectile motion, the vertical position will be given as a function of the horizontal position. If experimental data match the function, we gain confidence in its reliability. Alternatively, and more theoretically, we may consider the horizontal position to be a function of a parameter, $t$, to the first power and the vertical position to be a function of the same parameter to the second power. Then the horizontal and vertical positions are related by a second order, linear equation, which is the definition of a parabola. 
(d) let’s say the ball hits the wall 1/4 of its diameter lower than it’s initial height (this would be hard to see). If the ball has an 8 cm diameter, that means a fall of 2 cm, which, given, $\Delta y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2\Delta y}{g}} \approx 0.06$ s. If the wall is 6 cm away, say, the velocity is 100 ms$^{-1}$ or over 200 mhr$^{-1}$. This will put a good size hole in sheet rock. 
(e) see figure, pink curve. Free fall acceleration the same for all objects regardless of mass, so trajectories the same for same initial velocity.

5. (a) close system, so $\Delta p_A = -\Delta p_B \Rightarrow m_A\Delta v_A = -m_B\Delta v_B \Rightarrow \Delta v_A/\Delta v_B = -m_B/m_A$, so velocity change inversely related to mass number. 
(b) Say system initially at rest and isolated: $p_{\text{tot}} = 0 \Rightarrow m_s v_s - mv = 0 \Rightarrow m = \frac{v}{v_s} m_s$, if we call the standard mass’s direction positive. 
(c) Initial momentum zero, stays zero after firing; ball’s mass much smaller than cannon’s so it’s velocity much larger. 
(d) force same on both objects in each case, by Third Law. By Second Law, for a given force the mass number and acceleration are inversely related: smaller mass means larger acceleration, consequently, larger velocity after interaction.

6. (a) smaller; second half average translational velocity greater than first half. 
(b) double; acceleration constant so the square of the translational velocity increases linearly with distance $v^2 \propto a\Delta s$. 
(c) double; $v = v_t \propto \omega$ 
(d) double; sum of previous results. 
(e) no difference, only acceleration will be smaller, but ratios the same. 

7. Assume each story is 4 meters, and the person’s mass number is 50 kg. Assume further that “jumping” means free falling from rest. Set the positive direction “down”. Use $g = 10$ ms$^{-2}$. Set the level of the net to be the zero of the potential energy.
(a) \( v^2 = 2g\Delta y \Rightarrow v = \sqrt{2g\Delta y} = \sqrt{1600} = 40 \text{ ms}^{-1} \).
(b) \( ma\Delta y = mg\Delta y = 40000 \text{ J} = 40 \text{ kJ} \).

8. The absence of an arrow means the magnitude of the vector is zero.