Measurement Uncertainties Solution Set

1.
\[ \bar{x} = 9.75 \]
\[ \sigma = 0.68 \text{ [population standard deviation, i.e. } \frac{1}{n}] \]
\[ s = 0.72 \text{ [sample standard deviation, i.e. } \frac{1}{n-1}] \]

2.
\[ 0.08 \text{ m} \div 0.62 \text{ m} \times 100 = 13\% \]
\[ 0.4 \text{ m/s} \div 1.8 \text{ m/s} \times 100 = 22\% \]
\[ 2. \text{ km} \div 11. \text{ km} \times 100 = 18\% \]

3.
\[ 23.41 \text{ m} \times 10\% \div 100 = 2.34 \text{ m} \]
\[ 4.4 \text{ m/s} \times 0.4\% \div 100 = 0.0(2) \text{ m/s} \]
\[ 8.2 \text{ s} \times 8.2\% \div 100 = 0.7 \text{ s} \]

4.
\[ a = \frac{x}{2t^2} \Rightarrow x = \frac{1}{2} at^2 \]
\[ \frac{\partial x}{\partial a} = \frac{t^2}{2}; \quad \frac{\partial x}{\partial t} = at \]
\[ \sigma_x = \sqrt{\left(\frac{\partial x}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial x}{\partial t}\right)^2 \sigma_t^2} \]
\[ \%\sigma_x = \frac{\sigma_x}{x} \times 100 \]
\[ = \sqrt{\frac{t^4}{4} \sigma_a^2 + a^2 t^2 \sigma_t^2} \]
\[ = \sqrt{\frac{\sigma_a^2}{a^2} + 4 \frac{\sigma_a^2}{t^2}} \quad 9.4\% \]

5.
\[ N_\sigma = \frac{|\text{theoretical} - \text{experimental}|}{\sigma} = 2.3 \]
%\text{uncertainty} = \frac{\sigma}{\text{result}} = 6.7\% \\
%\text{difference} = \left| \frac{\text{theoretical} - \text{experimental}}{\text{theoretical}} \right| \times 100 = 14\%
6. \\
\sigma_z = \sqrt{\left( \frac{\partial z}{\partial \alpha} \right)^2 \sigma_\alpha^2 + \left( \frac{\partial z}{\partial t} \right)^2 \sigma_t^2} \\
= \sqrt{t^2 e^{2\alpha t} \sigma_\alpha^2 + \alpha^2 e^{2\alpha t} \sigma_t^2} \\
= e^{\alpha t} \sqrt{t^2 \sigma_\alpha^2 + \alpha^2 \sigma_t^2} \\
= 96
7. \\
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2} \\
= \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i^2 - 2x_i \bar{x} + \bar{x}^2)} \\
= \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N} - \frac{2 \bar{x} \sum_{i=1}^{N} x_i}{N} + \frac{\bar{x}^2 \sum_{i=1}^{N} 1}{N}} \\
= \sqrt{\bar{x}^2 - 2\bar{x}^2 + \bar{x}^2} \\
= \sqrt{\bar{x}^2 - \bar{x}^2}
8. \\
\frac{\partial z}{\partial x} = \frac{1}{y}, \quad \frac{\partial z}{\partial y} = \frac{-x}{y^2} \\
\sigma_z^2 = \left( \frac{\partial z}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial z}{\partial y} \right)^2 \sigma_y^2 \\
\frac{\sigma_z^2}{\sigma_x^2} = \left( \frac{\partial z}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial z}{\partial y} \right)^2 \sigma_y^2 \\
= \frac{1}{y^2} \sigma_x^2 + \frac{x^2}{y^2} \sigma_y^2
\[ \Rightarrow \%\sigma_z^2 = \%\sigma_x^2 + \%\sigma_y^2 \]

9.

\[ S_1 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} = 213 \]
\[ S_x = \sum_{i=1}^{N} \frac{x_i}{\sigma_i^2} = 3028 \]
\[ S_y = \sum_{i=1}^{N} \frac{y_i}{\sigma_i^2} = 9568 \]
\[ S_{xx} = \sum_{i=1}^{N} \frac{x_i^2}{\sigma_i^2} = 51717 \]
\[ S_{xy} = \sum_{i=1}^{N} \frac{x_i y_i}{\sigma_i^2} = 163970 \]

\[ m = \frac{S_{xy}S_1 - S_xS_y}{S_{xx}S_1 - S_x^2} = 3.22 \]
\[ b = \frac{S_{xx}S_y - S_xS_{xy}}{S_{xx}S_1 - S_x^2} = -0.88 \]
\[ m = \frac{S_1}{S_{xx}} = 0.01 \]
\[ m = \frac{S_{xx}}{S_{xx}S_1 - S_x^2} = 0.17 \]