Standing Waves on a String Solutions

Physical Concepts

1. What is the name given to a point on a vibrating string at which the displacement is always zero? What is the name given to a point at which the displacement is always maximum?
   node; anti-node

2. What are the conditions (with respect to the points of zero amplitude and maximum amplitude) that must hold in order to produce a standing wave on a vibrating string?
   The string characteristics and driving frequency must be such that integer values of the resulting wavelength fit completely within the length of the string; i.e., there is a quarter wavelength separating nodes and anti-nodes, where an anti-node must be at the fixed point(s) of the string.

3. How is the length of the string related to the wavelength for standing waves?
   \[ L = \frac{n\lambda}{2} \]

4. What is the longest possible wavelength for a standing wave in terms of the string length?
   \[ 2L \]

5. What would be the effect if the string stretched significantly as the tension increased? How would that have affected the data?
   The linear mass density decreases. If \( L \) remains fixed, then the combined effects of increased tension and decreased linear mass density results in the wave velocity increasing, which, for a given driving frequency, means a longer resulting wavelength. In any case, resonance requires higher frequencies than it otherwise would.

6. How is wave speed related to frequency and wavelength? How is the period of oscillation related to wave speed?
   \[ v = f\lambda; \ T \propto 1/v \]

7. How does the wavelength of a standing wave in a vibrating string vary as the tension in the string varies? As the linear mass density of the string varies?
   \[ v = \sqrt{F_T/\mu} \]

8. The velocity of a portion of rope at an instant when transverse waves are passing through it is the superposition of the velocities of the waves. Is the kinetic energy of a portion of the rope the superposition of the kinetic energies of waves passing through that portion? Justify your answer.
No, kinetic energy is a positive-definite scale and therefore cannot superpose.

9. Stringed musical instruments, such as violins and guitars, use stretched strings. Explain

(a) how tightening and loosening the string tunes them to their designated tone pitch or frequency;
   For fixed $L$, resonant $\lambda$ is fixed; $\sqrt{FT} \propto v \propto f$

(b) why the strings of lower tones are thicker or heavier;
   Thicker/heavier strings have greater linear mass density, which is related to velocity and therefore frequency as $1/\sqrt{\mu} \propto v \propto f$, so the greater $\mu$, the smaller $f$.

(c) why notes of higher pitch or frequency are produced when fingers are placed on the strings.
   For a given string, the wave velocity is set. Placing a finger on a string changes the effective length of the string and $L = n\lambda/2$. If $L$ is shortened, then so is the resonant wavelength. Since $v = f\lambda$, if $\lambda$ gets smaller, then $f$ must get bigger.