\[ \langle \Psi | \Psi \rangle = \langle \sum_{n} a_n^* | \sum_{m} a_m | \varphi_n | \varphi_m \rangle = \sum_{n} \sum_{m} a_n^* a_m \langle \varphi_n | \varphi_m \rangle = \sum_{n} |a_n|^2 \]

\[ \hat{H} = -\frac{\hbar^2}{2m} \nabla^2 - e/r^2 \]

\[ l = 0 \]

\[ r \to \infty \quad \Psi_{\text{rad}} \to 0 \]

\[ r \to 0 \quad \Psi_{\text{gen}} \to r^l = \text{constant (no nodes)} \]

**Perturbation theory**

\[ \hat{H} = \hat{H}_0 + \hat{H}_1 \]

\[ \hat{H}_0 | n \rangle = E_n | n \rangle \]

\[ \hat{H}_1 | n \rangle = E_1 | n \rangle \]

\[ \hat{H}_0: \text{unperturbed Hamiltonian} \]

\[ \hat{H}_1: \text{perturbation} \]

Expand in power series of \( \lambda \):

\[ | n \rangle = | n_0 \rangle + \lambda | n_1 \rangle + \lambda^2 | n_2 \rangle + \cdots \]

\[ E_n = E_{n_0} + \lambda E_{n_1} + \lambda^2 E_{n_2} + \cdots \]

\[ (\hat{H}_0 + \lambda \hat{H}_1)(| n_0 \rangle + \lambda | n_1 \rangle + \lambda^2 | n_2 \rangle + \cdots) = (E_{n_0} + \lambda E_{n_1} + \lambda^2 E_{n_2} + \cdots)(| n_0 \rangle + \lambda | n_1 \rangle + \lambda^2 | n_2 \rangle + \cdots) \]

\[ \hat{H}_0 | n_0 \rangle + \lambda (\hat{H}_1 | n_0 \rangle + \hat{H}_0 | n_0 \rangle) + \lambda^2 (\hat{H}_1 | n_0 \rangle + \hat{H}_0 | n_0 \rangle + \cdots \]

\[ = E_{n_0} | n_0 \rangle + \lambda (E_{n_0} | n_1 \rangle + E_{n_0} | n_0 \rangle) + \lambda^2 (E_{n_0} | n_2 \rangle + E_{n_0} | n_1 \rangle + E_{n_0} | n_0 \rangle) + \cdots \]

\[ \hat{H}_0 | n_0 \rangle = E_{n_0} | n_0 \rangle \]

\[ \hat{H}_1 | n_0 \rangle = E_{n_0} | n_0 \rangle \]

\[ \Rightarrow E_n = \langle n_0 | \hat{H}_1 | n_0 \rangle \]

In regards to 12.1:

Operators in different spaces always commute.

i.e. \( \hat{A} \text{ something}, \hat{B} \text{ something} \)

i.e. \( x, y, z, r, \theta, \phi \)

\( \hat{A} \) operators are constant matrices, w.r.t. the variables affected by \( \hat{B} \).

Note:

If operators commute \( \Rightarrow \) they share common eigenfunctions.

If operators share common eigenfunctions, they don't necessarily commute.

In regards to 13.2.1:

The behavior of the state function in this problem will be different than the one described in the book (in §11.9), b/c

the state function in this problem started out in an arbitrary state, not necessarily an eigenstate (i.e. \( \alpha \chi \)).