Postulates
\[ \hat{H} \Psi = i \hbar \frac{\partial \Psi}{\partial t} \]
\( \Psi(x,t) \) describes system
\( \hat{A} \rightarrow \hat{A} f = \alpha f \) \( \forall f \) \( \alpha \) set of eigenfunctions
\[ \Psi = \sum_n b_n f_n, \quad |\Psi> = \sum_n b_n |f_n> \]
\( |b_n|^2 \) = probability
Hermitian: \( \hat{A} = \hat{A}^\dagger \)
\[ \Psi \rightarrow f_n \]
\[ \hat{A} \Psi(\vec{r},\vec{p}) = E \Psi(\vec{r},\vec{p}) \rightarrow \Psi_{lm}(\vec{r}) = e^{ik \cdot \vec{r}} \]
\[ \phi(\vec{r}) = \phi_{lm}(\vec{r}) = R_n(l) Y_{lm}(\theta, \phi) \]
\[ [\hat{\mathbf{A}}, \hat{\mathbf{J}}] = [\hat{\mathbf{J}}, \hat{\mathbf{J}}^2] = 0 = [\hat{\mathbf{J}}^2, \hat{\mathbf{J}}^2] \]
\[ \hat{J}_m |l m> = \tau_j (l+1) |l m> \]
\[ \hat{J}_m (\theta, \phi) = \tau_j (l+1) Y_{lm}(\theta, \phi) \]
There are operators which have only matrix representations
\[ \{ \phi_n^2 \rightarrow |\Psi> = \sum_n a_n |0_n> \quad a_n = <\phi_n | \Psi> \] vectors must be represented in the same basis in order for them add, subtract, or whatever
it's the same in Hilbert space
coefficients are always associated with a specific representation
\[ \hat{A} |\Psi> = a |\Psi> \]
\[ \{ |\Psi> = \sum_n a_n |\phi_n> \rightarrow \sum_n a_n \hat{A} |\phi_n> = \sum_n b_n |\phi_n> \]
\[ \{ |\Psi> = \sum_n a_n |\phi_n> \rightarrow \sum_n a_n \hat{A} |\phi_n> = \sum_n b_n |\phi_n> \]
\[ \sum_n \bar{a}_n a_m = b_k \]
\[ A_{km} = <\phi_k | \hat{A} | \phi_m> \]
\[ \hat{\mathbf{J}} \rightarrow \sum l m \rightarrow |\Psi> = \sum_n a_n |\phi_n> \quad a_n = <\phi_n | \Psi> \]
\[ \| |\Psi> \|^2 = <\Psi | \Psi> = \sum_n |a_n|^2 = \sum_n \sum_m \bar{a}_m a_n <\phi_m | \phi_n> = \sum_n |a_n|^2 \]
\[ <\Psi | \Psi> = \sum_n \bar{a}_n a_n \]
\[ \sum_n |\phi_n> <\phi_n | = I \] review of matrix multiplication
\[ \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} \]
in general, matrices do not commute
continued on next page