Experiment 2
LRC Circuit Steady State Response

1 Purpose

In this experiment we will investigate steady state current and voltages in an LRC circuit which is driven at frequencies equal to, above and below the natural oscillation frequency of the circuit. Specifically, the current as a function of driving frequency and the phase difference between current and applied voltage will be investigated. In addition, the voltage drop across L at resonance, will be studied.

2 Predictions

For a series LRC circuit, driven by a sinusoidal voltage oscillator of frequency $f$ and amplitude $V_0$, it is shown in most textbooks that

1. The amplitude of the current is

$$I_0 = \frac{V_0}{\sqrt{(2\pi f L - \frac{1}{2\pi f C})^2 + R^2}} \quad (1)$$

Recall that the amplitude and rms value for a sinusoidal oscillation are related by $V_{rms} = V_0/\sqrt{2}$ So Equation 1 becomes

$$I_{rms} = \frac{V_{rms}}{\sqrt{(2\pi f L - \frac{1}{2\pi f C})^2 + R^2}} \quad (2)$$

2. The natural frequency of the circuit is

$$f_0 = \frac{1}{2\pi \sqrt{LC}} \quad (3)$$

3. The phase angle $\phi$ between the current and the driver voltage is

$$\tan \phi = \frac{(2\pi f L - \frac{1}{2\pi f C})}{R} \quad (4)$$
3 Apparatus

The circuit to be studied is a series LRC circuit with a sinusoidal driving voltage source. The circuit is shown in Figure 1 with “ideal” components.

Because the inductor has a finite resistance, and the waveform generator has an internal resistance, the actual circuit is that of Figure 2.

In the equations above, the quantity $R$ represents the total series resistance. But the voltage drop across the 2.2 $K\Omega$ resistor is just $I(2200.)$. 
4 Procedures

4.1 Basic Measurements

1. Connect the circuit shown in Figure 2. Channel 2 of the oscilloscope measures the driving voltage, \( V \), from the sinusoidal output of the signal generator. Channel 1 measures the voltage drop (\( V_R \)) across the resistor. Note that both channels of the scope have the same ground. This is also the ground of the waveform generator.

2. Change the signal generator frequency until \( V_R \) is at a maximum. This is the resonance frequency.

3. Use the Measure menu to set up measurements of the rms voltages, and the frequency. Set the waveform generator output to be about 1 volt. Then measure \( V_{R\text{rms}} \), the rms value of \( V_R \) and \( V_{\text{rms}} \), the rms value of \( V \). Also, use the time cursors to measure the time difference, \( \Delta t \), between adjacent crossings of zero

\[
\Delta t = t_{V_R=0} - t_{V=0}
\]

Note that \( \Delta t \) will be both negative and positive. Figure 3 shows an example plot of \( V \) and \( V_R \).

![Figure 3: V and V_R at a frequency above the resonance frequency.](image)

4. Do this for 8 – 10 frequencies between 100 Hz and 4000 Hz; the resonance frequency should be one of these. Put these values into a spreadsheet, and compute the phase difference, \( \phi \), between current and voltage.

You can find the phase difference with the equation

\[
\phi = 2\pi f \Delta t
\]
Make plots of $V_{Rrms}$ vs. frequency and $\tan \phi$ vs. frequency. The plots should include both measurements and predictions of $V_{Rrms}$ and $\tan \phi$. The measurement of $V_{Rrms}$ is what is obtained from the scope. The prediction of $V_{Rrms}$ is the predicted $I_{Rrms}$ Equation 2 times $R_{load}$ (2.2K). The measurement of $\tan \phi$ is obtained from $\phi$ obtained from Equation 5. The prediction of $\tan \phi$ is obtained from Equation 4. (In computing the predicted value of $\phi$, you may neglect the internal resistance of the waveform generator.)

4.2 Comparing $V_L$ and $V$

1. To compare the waveforms of the voltage across the inductor, $V_L$, and the driving voltage, $V$ at resonance, it is necessary to change the circuit so that the inductor has one side at ground. This is because the oscilloscopes we use measure voltage between the input and ground. First set the waveform generator to the resonance frequency. Set up the circuit shown in Figure 4.

2. Read out the waveforms, and paste them into Excel. Make a plot of the two.

3. Questions: Do $V_L$ and $V$ have the correct phase difference? Is the voltage drop across the inductor larger than the input driving voltage? Given $\omega_0$ and $L$, for what values of $R$ will $V_L$ be greater than $V_0$?