9 Torque

Introduction
In Part I of this lab, you will observe static equilibrium for a meter stick suspended horizontally. In Part II, you will observe the relationship between torque, moment of inertia and angular acceleration for a rotating rod with two masses on either end. You will vary the mass connected (and therefore the torque applied) to the rod by two pulleys. You will also change the moment of inertia of the rod system by changing the distance of the masses from the center of mass of the rod.

Materials
Part I: meter stick apparatus, string, force sensors, Data Studio, masses
Part II: Rotary Motion sensor, Data Studio, pulley, adjustable bar, masses, string and meter stick

Reference
Giancoli, Physics 6th edition: Chapter 8, sections 4,5,6; Chapter 9, sections 1,2

Theory
Part I: When forces act on an extended body, torques, or rotations about axes on the body, can result as well as translational motion from unbalanced forces. Static equilibrium occurs when the net force and the net torque are both equal to zero. We will examine a special case where forces are only acting in the vertical direction and can therefore be summed simply without breaking them into components:

\[ F_{net} = F_1 + F_2 + F_3 + ... = 0 \] (1)

Torques may be calculated about the axis of your choosing and because a meter stick is our body, the length of the lever arm can be conveniently determined without resolving components.

\[ \tau_{net} = \tau_1 + \tau_2 + \tau_3 + ... = 0 \] (2)

Each torque is specified by the equation:

\[ \tau = Fd \] (3)

Normally, up is "+" and down is "-" for forces. For torques, it is convenient to define clockwise as "-" and counterclockwise as "+". Whatever you decide to do, be consistent with your signs and make sure you understand what a "+" or "-" value for your force or torque means directionally.
Part II: Moment of inertia is a measure of the distribution of mass in a body and how difficult that body is to accelerate angularly. Refer to Figure 9.2. The moment of inertia is the sum of the moments of inertia of the two masses and the rod. For the masses that slip onto the rod, we will assume point masses, thus the moment of inertia for one of the two masses is:

\[ I_{\text{mass}} = mr^2 \]  (4)

Where \( r \) is the distance of the center of mass from the axis of rotation located at the center of the rod. Because the masses can be moved along the rod, \( r \) will be adjusted to change their moment of inertia. The moment of the inertia of the rod with mass \( M \) and a length \( L \):

\[ I_{\text{rod}} = \frac{1}{12} M_{\text{rod}} L^2 \]  (5)

The moment of inertia for a rod with length \( L \) and two masses on each end at a distance \( r \) is simply the sum of the components as defined by Equations 4 and 5:

\[ I = 2mr^2 + \frac{1}{12} M_{\text{rod}} L^2 \]  (6)

The second term is multiplied by two because there are two point masses. Given the moment of inertia of the entire system, \( I \), and the torque, \( \tau \), applied by the mass \( M \) hanging from the pulley, angular acceleration, \( \alpha \) can be found. If the force is held constant, varying the distance of the masses from the center of mass will change the angular acceleration accordingly. Just as \( F = ma \) for translational motion,

\[ \tau = I\alpha \]  (7)

Torque, \( \tau \), is an applied force that causes rotation with an angular acceleration, \( \alpha \), in an object with a moment of inertia, \( I \). The rod is attached at its center to a rotary motion sensor which has a string wrapped around it. The string has a mass tied to one end (that will vary) and is laid over an additional pulley which allows the falling motion of the mass to be converted to a torque on the rotary motion sensor. For the purposes of this lab, the applied torque will be due to a mass accelerated by gravity acting on the pulley of the rotary motion sensor. This is actually an approximation, see the Appendix for a full derivation of the applied torque. We will use the approximation \( \tau \approx MgR \), where \( M \) is the mass hanging on the pulley and \( R \) is the radius of the Rotary Motion pulley.

By adjusting the masses on the rod, we can observe how an increased moment of inertia (where either the mass is distributed farther from the center of mass or the total mass is increased) will result in a decreased angular acceleration for the same torque. It is the same type of relationship as the one you observed for Newton's Second Law.

Procedure
Part I: Static Equilibrium

1. Weigh the meter stick you use. Record the mass and estimate the uncertainty.

2. Set up the meter stick and force sensors as shown in Figure 9.1. The meter stick will be suspended from a beam via the two force sensors. (These will also be used to determine the upward vertical forces at these positions.) The force sensors must be attached vertically anywhere that yields equilibrium by string tied tightly around the meter stick.

3. Attach three masses to the meter stick using string. Neglect the masses of the support strings.

4. Attach the force sensor cords to the Interface box as you have done in previous labs.

5. For today's lab you do NOT need to graph the force sensors over time, instead, drag the force icons to the Digits icon in the "Displays" menu. This will give you a digital readout of the force sensor data at the given time. Create two Digit displays, one for each force sensor on the meter stick.

6. Then hang a known weight on each force sensor to verify that it is reading correctly. If you need to increase the precision of the display, select the downward-facing arrow on the icon that says "3.14" and select Increase Precision. This will increase the number of significant figures. Next, tare each force sensor without weight to establish zero.
7. Hang the masses on the three remaining strings and balance your system by moving the three weights and watching the Digits displays. All forces must be vertical to avoid difficulties, so make sure the meterstick is level and be sure the force sensors are pulling straight upward.

8. Record the position and mass on the meterstick for each mass.

9. Using the following data sheet to record the results, calculate the sum of the masses responsible for the positive forces and the sum of those responsible for the negative forces. (Forces 5 and 6 are the force meters.) Check to see if the sums are equal within calculated uncertainties.

<table>
<thead>
<tr>
<th>Mass (g) ± g</th>
<th>Force #</th>
<th>Force (N) ± N</th>
<th>Position (m) ± m</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 = )</td>
<td>F1</td>
<td>( m_1 \times g = )</td>
<td></td>
</tr>
<tr>
<td>( m_2 = )</td>
<td>F2</td>
<td>( m_2 \times g = )</td>
<td></td>
</tr>
<tr>
<td>( m_3 = )</td>
<td>F3</td>
<td>( m_3 \times g = )</td>
<td></td>
</tr>
<tr>
<td>( m_4 = )</td>
<td>F4</td>
<td>( (m_4 \text{meterstick}) \times g = )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>F6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10. Using the zero position (\( x = 0 \) m) of the meter stick as the axis of rotation and counterclockwise torques as positive, determine the sum of the torques acting in both directions and record them on the data sheet. Check for equality between positive and negative sums within the calculated uncertainties. Now imagine the lever arm is located at the axis point in the middle of the meter stick (\( x = 0.50 \) m) and recalculate torques. Check for equality between positive and negative results, within calculated uncertainties.

11. Make a copy of the tables you have developed. Then alter the arrangements of masses and force sensors and edit the copied table and see if the new configuration satisfies the conditions for equilibrium.

NOTE: here the lever arm = (force position - axis position). If signs are carefully kept, the sign of the force times the lever arm sign will successfully give the sign of the resulting torque.
## Torque Calculation Table

<table>
<thead>
<tr>
<th></th>
<th>Lever Arm (m) Axis at $x=0\text{m}$</th>
<th>Torque (N-m) Axis at $x=0\text{m}$</th>
<th>Lever Arm (m) Axis at $x=0.5\text{m}$</th>
<th>Torque (N-m) Axis at $x=0.5\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of Positive Torques</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of Negative Torques</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Difference</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sum of all Torques</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Part II: Motion with Constant Angular Acceleration

The experiment uses a mass hanging on a string over a pulley to exert a constant torque on the system resulting in a constant angular acceleration on the system.

1. Set up the apparatus as shown in Figure 9.2, make sure to connect the rotary motions sensor to the interface box.

![Figure 9.2](image-url)
2. Make sure that the pulley is set up to give positive values for angular position. This means that the rotary motion sensor will turn in a counterclockwise direction as the mass on the pulley drops. (The motion sensor may have an indication of which direction is positive taped to it.)

3. Adjust the measurement rate to 10 Hz on the **Rotary Motion Sensor**.

4. Measure angular position, \( \theta \), in radians, angular velocity, \( \omega \), in radians/s and angular acceleration, \( \alpha \), in radians/s². Create graphs of these quantities vs. time.

5. Measure and record the radius of the rotary motion sensor pulley around which the string is wound. Hang a mass of 20 grams from the pulley, press **Start**, and release it.

6. Press **Stop** just before the hanging mass reaches the table. You will repeat this for five trials with different masses and create a table with the following format:

<table>
<thead>
<tr>
<th>Run</th>
<th>( R ) (m)</th>
<th>( M ) (kg)</th>
<th>( I_{masses} = 2mr^2 )</th>
<th>( I = 2mr^2 + I_{rod} )</th>
<th>( \alpha )</th>
<th>( \tau = I\alpha )</th>
<th>( \tau = MgR )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = r_0 )</td>
<td>0.02</td>
<td></td>
<td>( I_{masses} = 2mr^2 )</td>
<td>( I = 2mr^2 + I_{rod} )</td>
<td>( \alpha )</td>
<td>( \tau = I\alpha )</td>
<td>( \tau = MgR )</td>
</tr>
<tr>
<td>( r = r_0 )</td>
<td>0.04</td>
<td></td>
<td>( I_{masses} = 2mr^2 )</td>
<td>( I = 2mr^2 + I_{rod} )</td>
<td>( \alpha )</td>
<td>( \tau = I\alpha )</td>
<td>( \tau = MgR )</td>
</tr>
<tr>
<td>( r = r_0 )</td>
<td>0.06</td>
<td></td>
<td>( I_{masses} = 2mr^2 )</td>
<td>( I = 2mr^2 + I_{rod} )</td>
<td>( \alpha )</td>
<td>( \tau = I\alpha )</td>
<td>( \tau = MgR )</td>
</tr>
<tr>
<td>( r = .8r_0 )</td>
<td>0.06</td>
<td></td>
<td>( I_{masses} = 2mr^2 )</td>
<td>( I = 2mr^2 + I_{rod} )</td>
<td>( \alpha )</td>
<td>( \tau = I\alpha )</td>
<td>( \tau = MgR )</td>
</tr>
<tr>
<td>( r = 0.7r_0 )</td>
<td>0.06</td>
<td></td>
<td>( I_{masses} = 2mr^2 )</td>
<td>( I = 2mr^2 + I_{rod} )</td>
<td>( \alpha )</td>
<td>( \tau = I\alpha )</td>
<td>( \tau = MgR )</td>
</tr>
</tbody>
</table>

**IMPORTANT**: The difference between \( M \) and \( m \): the first is the mass on the string which accelerates the rod, the second represents the point masses on each end of the rod. Another important distinction: \( r \) is the distance between the center of mass of the point mass and the axis of rotation (the center of the rod), \( R \) is the radius of the pulley on the rotary motion sensor around which the string is wrapped.

**For your Lab Report:**
Include a sample calculation of the torques in the last two columns from Part II: \( \tau = I\alpha \) and \( \tau = MgR \). Estimate the uncertainty of the measurements for the masses of the point masses, the distance from the axis of rotation \( (r) \), and the angular acceleration as recorded by Data Studio. Use these estimates to propagate your uncertainty.
Appendix

The calculation of the torque applied to the rotary motion sensor in Part II is as follows:

The rotary motion sensor has a radius, $R$, which is the distance from the axis of rotation at which the force, $T$, acts. $T$ is the tension in the string attached to the pulley and is due in part to the weight on the string, $Mg$, where $M$ is the mass of the hanging mass and $g$ is the acceleration due to gravity. Figure 9.3 below shows a rough sketch of the set up.

![Figure 9.3](image)

The application of Newton's Second Law to the hanging mass gives:

\[ \Sigma F_y = Ma \]
\[ \Sigma F_y = Mg - T = Ma \]

The downward direction is taken to be positive. We can substitute for the linear acceleration, $a$:

\[ a = R\alpha \]

Where $\alpha$ and $R$ are the angular acceleration and the radius of the rotary motion sensor, respectively. Continuing to solve the equation gives us:

\[ Mg - T = MR\alpha \]
\[ T = Mg - MR\alpha \]

$T$ is the tension in the string, it is a force not a torque, to find the torque acting on the pulley, we must multiply by the distance from the axis of rotation at which the force acts:

\[ \tau = RT \]
\[ \tau = MgR - MR^2\alpha \]
For the set up of this experiment, take the following example where $M=0.06$ kg, $R \approx 0.025$ m and $\alpha \approx 2.58$ rad/sec$^2$. Substituting these values, gives:

$$MgR = 14.7 \times 10^{-3} Nm$$
$$MR^2\alpha = 0.0969 \times 10^{-3} Nm$$

From this example, we see that $MgR \gg MR^2\alpha$ so we can approximate $\tau = MgR$. 