Measurement Uncertainties Lab

Introduction
In Part I of this exercise you will measure the mass and dimensions of a solid sphere of steel. From your measurements, you will calculate the density of the sphere. You will have to take multiple measurements of the diameter of the sphere in order to calculate the uncertainty in diameter and the corresponding uncertainty in volume.

For your Lab Report
This experiment does not require a normal lab report. In writing up Part I, give a one paragraph summary of your results, and include in a sample calculation of the density of steel and its uncertainty. Most of your grade will be determined by your performance on the problems in Part II. In your answers, be sure to show your work.

Materials
Part I: steel sphere, scale, meter stick, Excel

Reference
Measurement Uncertainties handout.
Giancoli, Physics 6th Edition: Chapter 1, sections 5

Theory
Part I: The density (symbolized by the Greek letter, $\rho$ or “rho”) of an object is defined as the mass divided by the volume:

$$\rho = \frac{m}{V}$$

The volume of a sphere is dependent on its radius, $r$, or its diameter, $d$:

$$V = \frac{4\pi}{3} r^3 = \frac{\pi}{6} d^3 \quad (1)$$
Procedure

**Part I:** Measurement of the density of a steel sphere.

1. Determine the value of and the uncertainty in diameter, \(d\), of the steel sphere by measuring it at least 10 times with a meter stick. The meter stick is used deliberately in order to have a large uncertainty.

2. Calculate the average diameter and standard deviation. To use Excel for the calculation, enter the diameter values in cells A2 to A11. In cell A12 enter "=average(A2:A11)" and in cell A13 enter "=stdev(A2:A11)".

3. Calculate the density of steel and its uncertainty. Convert the results to kg/m\(^3\). Within how many standard deviations does your result agree with the accepted density of steel?

\[
\left(7.8 \times 10^3 \text{ kg/m}^3\right)
\]

4. Write a short conclusion.

**Part II:** Problems

1. A certain device has made 10 measurements of a quantity \(x\): 6.46, 6.77, 6.54, 6.03, 6.65, 6.40, 6.88, 6.35, 6.51 and 6.27. Compute the average value of \(x\) and \(x\sigma\) using Eqn 1.2 of the Measurement Uncertainties handout. Then repeat the calculation in Excel with the special functions used in Part I.

2. Find the percentage uncertainty for the values with given uncertainties below.

<table>
<thead>
<tr>
<th>Value</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.3 m</td>
<td>± 0.4 m</td>
</tr>
<tr>
<td>9.2 m/s</td>
<td>± 0.5 m/s</td>
</tr>
<tr>
<td>23.28 s</td>
<td>± 0.55 s</td>
</tr>
</tbody>
</table>

3. Find the absolute uncertainty for the values with given percentage uncertainties below.

<table>
<thead>
<tr>
<th>Value</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.21 m</td>
<td>± 10 %</td>
</tr>
<tr>
<td>12.0 m/s</td>
<td>± 0.8 %</td>
</tr>
<tr>
<td>8.59 s</td>
<td>± 4 %</td>
</tr>
</tbody>
</table>

4. Length \(A\) is measured to be 4.5 cm ± 0.3 cm and length \(B\) is 4.0 cm ± 0.7 cm. What is the difference between the two lengths, \(D = A - B\), and the absolute uncertainty in \(D\)? Calculate the percent uncertainty of \(A\), \(B\) and their difference. Is the percent uncertainty of the \(D\) much different than that of either \(A\) or \(B\)? If the uncertainty in length \(A\) is reduced from 0.3 cm to
0.1 cm, a factor of 3 improvement, what is the new uncertainty in the difference of \( A-B \)? Can you explain the results?

5. Assuming \( x, t, \) and \( a \) are related by the equation \( a = \frac{2x}{t^2} \), find the value for \( \% \sigma_a \) if \( x = 10.0 \text{ m} \pm 2.0 \text{ m} \) and \( t = 8.00 \text{ s} \pm 0.30 \text{ s} \).

6. An experimental measurement has a result \( 7.95 \pm 0.01 \), and a predicted value of \( 8.00 \). How many standard deviations is the experimental measurement from the predicted value? What is the percent uncertainty in the measurement? What is the percent difference between measured and predicted value? (ignore uncertainties in this calculation)

7. If the uncertainty of a single length measurement is \( \pm 0.5 \text{ cm} \), how many measurements, \( N \), of the same length are necessary so that the average of the measurements has an uncertainty of \( \pm 0.05 \text{ cm} \)?

8. If \( \theta = 25^\circ \pm 2^\circ \), what is the value of the uncertainty for \( \sin (\theta) \)? Show your work.