Introduction
In this lab, you will be observing the conservation of momentum under three different conditions. In Part I, you will observe the elastic collision of two dynamics carts with magnets that repel one another. In Part II, you will observe the perfectly inelastic collision of two dynamics carts with Velcro ends that attach on contact. In Part III, you will place two carts together and observe conservation of momentum as a spring propels the two in opposite directions from rest.

Materials
All Parts: Dynamics track, Data Studio, Motion Sensor, Excel
Part I: two dynamics carts with magnet ends
Part II: two dynamics carts with velcro ends
Part III: one dynamics cart with spring-loaded plunger

Reference
Giancoli, Physics 6th Edition: Chapter 7, sections: 1,2,4,5,6

Theory
The conservation of linear momentum \( \mathbf{p} = \mathbf{mv} \) is an important concept in physics. In a closed system, whenever momentum is conserved the initial and final total momentum are equal. By a "closed system" we mean that there are no outside forces acting on the system. For this lab, we will assume that there are no significant outside forces--such as friction--present during the collisions. This will not literally be the case but if the collision is short in duration, it is reasonable to make this assumption because the friction is small and the duration over which it acts is short. Therefore, \( \sum p_i = \sum p_f \). We will examine motion of two carts in only one dimension so the initial and final momentum equations are:

Initial: \( p_i = m_1v_{i1} + m_2v_{i2} \)

Final: \( p_f = m_1v_{f1} + m_2v_{f2} \)

And finally, conservation of momentum \( \sum p_i = \sum p_f \) gives

\[ m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2} \quad (1) \]

Part I: An elastic collision is one in which kinetic energy is conserved, \( KE_f = KE_i \) as well as momentum:

\[ KE_i = \frac{1}{2} (m_1v_{i1}^2 + m_2v_{i2}^2) \]
If there are no external forces acting on the objects in a collision (such as friction) the momentum is conserved, but the collision is not necessarily perfectly elastic. Kinetic energy must be conserved in order to define the collision as elastic. Kinetic energy is often dissipated through vibrations of each object upon collision. For this reason, we use similar poles of magnets (which repel one another) to prevent physical contact between the two carts during the elastic collision.

Part II: Collisions where \( KE_f < KE_i \) are referred to as inelastic collisions and the energy dissipated is \( \Delta KE = KE_i - KE_f \). A collision where the two objects stick together is referred to as a perfectly inelastic collision because the maximum possible energy is dissipated (typically in the form of heat). Momentum, however, is still conserved. You will observe this in the case of two carts which become attached upon collision by the velcro fabric on their ends.

Part III: You will also study conservation of momentum for two carts pushing away from each other through the force exerted by a coiled spring and plunger on one of the carts. Because the system is initially at rest and momentum is conserved, the final momentum of each of the two carts must be equal in magnitude and opposite in direction. The resulting total momentum of the system is equal to zero, the initial total momentum. We have \( m_1 v_1 = m_2 v_2 \) and thus

\[
\frac{v_1}{v_2} = \frac{m_2}{m_1}.
\]

(2)

The total energy of the system is conserved but kinetic energy is not because the total energy stored in the two cart system begins as elastic potential energy in the coiled spring. After the initial impulse when the spring is released, the energy is shared by the two carts in the form of kinetic energy. The kinetic energy is distributed between the two carts such that momentum is
conserved. They will NOT have the same kinetic energies unless the masses are equal. Using
the above equation and the equation for kinetic energy,

\[ \frac{KE_1}{KE_2} = \frac{\frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_2 v_2^2} = \frac{m_1}{m_2} \left( \frac{v_1}{v_2} \right)^2 = \frac{m_2}{m_1} \]  

(3)

And thus

\[ \frac{KE_1}{KE_2} = \frac{m_2}{m_1} \]  

(4)

Procedure
All three experiments will be conducted using the dynamics track with a motion sensor at each
end. Steps 1-5 of Part I apply to all three parts of the lab (steps 1-3 need only be performed once
at the beginning).

Part I: Elastic collisions

1. Attach the two motion sensors to the Data Studio interface. Double-click on each motion
sensor icon and set the trigger rate to 25 Hz. Set the switch on each motion sensor to the short-
range setting.

2. The figure below shows typical velocity vs. time data for a collision where \( m_2 \) is initially at
rest. Since the motion sensor for \( m_2 \) is on the opposite end of the track and oriented the opposite
direction, you will need to reverse the sign of \( v_2 \) in order to get the appropriate velocity relative
to the system. (In this graph, both values are recorded as negative and one should be made
positive.) This can be done in Data Studio but it is easier to reverse the sign manually by
multiplying \( v_2 \) results by -1 when you record them into Excel.

![Figure 7.2 Velocity of the two carts measured throughout the collision. The sign of the velocity of Cart 2 is reversed.](image)
Note: since there is actually some friction acting on the carts (as shown by the slight downward slope of the otherwise horizontal lines), it is important to determine the velocity from the graph immediately before and after the collision. (This way, the time over which friction has acted is minimal and therefore its effect on the conservation of momentum will be minimal.)

3. Orient the dynamics carts with the magnets toward each other. Since the magnets are strong you should keep your watch and any floppy disks away from them.

4. Create a table in Excel like the one below for your data on elastic collisions. In all cases \( m_1 \) is the incident cart and \( m_2 \) is stationary or moving slowly if you wish. (If you choose to have it moving slowly, make sure that you record its initial velocity under \( v_{2i} \). Remember that the sign of the velocity for \( m_2 \) has to be reversed.

<table>
<thead>
<tr>
<th>( m_1 ) Mass of cart</th>
<th>( m_2 ) Mass of cart</th>
<th>( v_{1i} )</th>
<th>( v_{2i} )</th>
<th>( v_{1f} )</th>
<th>( v_{2f} )</th>
<th>( P_i )</th>
<th>( P_f )</th>
<th>( %\Delta P )</th>
<th>( KE_i )</th>
<th>( KE_f )</th>
<th>( \Delta KE )</th>
<th>( %\Delta KE )</th>
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5. Measure the mass of the cart and record it into the table. The masses labeled as “+0.5 kg” are shorthand for you to add 0.5 kg to the cart and put the total mass of the cart plus 0.5 kg in the spreadsheet cell.

6. Run the trials listed in the table and graph them using Data Studio. (Press **Start**, accelerate the carts towards each other, record the collision, let them separate and press **Stop**.) Note: the magnets produce a collision that is approximately elastic when the carts are repelled solely by the magnetic repulsive force. Therefore, **your results will be most accurate if you avoid accelerating the carts so hard that they physically hit each other**.

7. Find the values for the initial and final velocities of both carts and record them in your Excel table. Use those values to calculate the values in the rest of the table.

8. Estimate the uncertainty in the initial and final momentum.
9. Determine if momentum is conserved. If the initial and final values of momentum are not equal, examine the change in momentum as a percent of the magnitude of the average momentum based on the initial and final values calculated. 

\[
\Delta P = P_f - P_i; \quad \%\Delta P = 100 \times \frac{\Delta P}{P}
\]

10. For each trial, use the value of \( \Delta KE \) you calculated to determine if the collision is elastic. If \( \Delta KE \) is not close to 0, look at its magnitude as a percentage of the magnitude of the average kinetic energy based on the initial and final values calculated. \( \%\Delta KE = 100 \times \frac{\Delta KE}{KE} \)

**Part II: Perfectly Inelastic Collisions**

1. Orient the two carts with their velcro ends are towards each other so that the carts will stick together in a collision.

2. Repeat the procedure from Part I using trials 1, 2 and 4 from the table. Add the data from Part II to the table below the data from Part I. Make it clear which data belongs to which experiment. Discuss the results.

3. Is momentum conserved? Look at the percent change in the momentum if not.

4. Is \( \Delta KE \) larger than for the inelastic collisions of Part I? What is the percent change?

**Part III: Conservation of Momentum for Two Carts Pushing Away from Each Other**

1. Place the two carts against each other with the plunger of the dynamics cart completely retracted and latched in its maximum position.

2. Press **Start** in Data Studio and then gently tap the plunger release button on the top of the cart so that the carts push each other apart.

3. Press **Stop** after the carts have reached the end of the track.

4. Record the final velocity of each of the carts and complete the first row of the table below. Since both the carts are initially at rest, only final velocity values are included in the table.

<table>
<thead>
<tr>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( v_1 )</th>
<th>( v_2 )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( m_2/m_1 )</th>
<th>( v_1/v_2 )</th>
<th>( KE_1 )</th>
<th>( KE_2 )</th>
<th>( KE_1/KE_2 )</th>
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<td>Mass of cart</td>
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5. Repeat the process for each of the different masses shown in the table above. Discuss the results.

6. Is momentum conserved?
7. Is kinetic energy shared according to the ratio of the masses as predicted by the theory? (See Equation (4))

For your Lab Report
Include sample calculations of the initial and final momenta from Part I, trial 1. Estimate the uncertainty for the mass and velocity measurements and include these in your calculation. Compare your results with uncertainties to see if momentum was conserved.