Give all the answers in terms of given quantities (constants such as $\varepsilon_0$, etc) may also appear in any solution). Work in a logical, clear and neat fashion. Work each problem on its own sheet of paper – extra paper will be provided if you should need it. No OPEN BOOK. No memory aids of any kind, electronic or otherwise, may be used. Partial credit will be awarded for incomplete solutions.

Good luck!

1. Two long coaxial solenoids each carry current $I$, but in opposite directions. The inner solenoid (radius $a$) has $n_1$ turns per unit length, and the outer one (radius $b$) has $n_2$. Find $\mathbf{B}$ in each of the three regions: (i) inside the inner solenoid; (ii) between them; and (iii) outside both.

   The field inside a solenoid is $\mu_0 n I$, and outside it is zero. The outer solenoid’s field points to the left ($-\hat{z}$), whereas the inner one points to the right ($+\hat{z}$). So: (i) $\mathbf{B} = \mu_0 n_1 n_2 \hat{z}$, (ii) $\mathbf{B} = -\mu_0 n_2 \hat{z}$, (iii) $\mathbf{B} = 0$.

2. Find the vector potential of a finite segment of straight wire, carrying a current $I$. [Put the wire on the z axis, from $z_1$ to $z_2$.]

   \[ \mathbf{A} = \frac{\mu_0}{4\pi} \int \frac{I}{s} \, ds = \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \frac{dz}{\sqrt{z^2 + s^2}} = \frac{\mu_0 I}{4\pi} \ln \left[ \frac{z_2 + \sqrt{z_2^2 + s^2}}{z_1 + \sqrt{z_1^2 + s^2}} \right] \hat{z} \]

   \[ \mathbf{B} = \nabla \times \mathbf{A} = -\frac{\partial A}{\partial s} \hat{\phi} = -\frac{\mu_0 I}{4\pi} \left[ \frac{1}{z_2 + \sqrt{z_2^2 + s^2}} - \frac{1}{z_1 + \sqrt{z_1^2 + s^2}} \right] \hat{\phi} \]

   or, since $\sin \theta_1 = \frac{z_1}{\sqrt{z_1^2 + s^2}}$ and $\sin \theta_2 = \frac{z_2}{\sqrt{z_2^2 + s^2}}$,

   \[ \mathbf{B} = \frac{\mu_0 I}{4\pi s} (\sin \theta_2 - \sin \theta_1) \, \hat{\phi} \] (as in Eq. 5.35).

3. Find the force of attraction between the two magnetic dipoles, $\mathbf{m}_1$ and $\mathbf{m}_2$, oriented as shown below, a distance $r$ apart

   \[ \mathbf{m}_1 \quad \text{m}_2 \]
Problem 9.3

(a) 

According to Eq. 6.2, \( F = 2\pi IR B \cos \theta \). But \( B = \frac{2\pi}{4\pi} \left[ (m_1 \cdot \hat{r} - m_2 \cdot \hat{r}) \right] \), and \( B \cos \theta = B \cdot \hat{y} \), so \( B \cos \theta = \frac{2\pi}{4\pi} \left[ (m_1 \cdot \hat{r})(\hat{r} \cdot \hat{y}) - (m_1 \cdot \hat{y}) \right] \). But \( m_1 \cdot \hat{y} = 0 \) and \( \hat{r} \cdot \hat{y} = \sin \phi \), while \( m_1 \cdot \hat{r} = m_1 \cos \theta \). Thus \( B \cos \theta = \frac{2\pi}{4\pi} 3m_1 \sin \phi \cos \phi \).

\[ F = 2\pi IR \frac{\sin \phi}{r} 3m_1 \sin \phi \cos \phi \]

Now \( \sin \phi = \frac{R}{r}, \cos \phi = \sqrt{r^2 - R^2}/r \), so \( F = 3\frac{\pi}{2} m_1 IR \sqrt{r^2 - R^2}/r \).

But \( IR^2 \pi = m_2 \), so \( F = \frac{3\pi}{2} m_1 m_2 \sqrt{r^2 - R^2}/r \), while for a dipole, \( R \ll r \), so \( F = \frac{3\pi}{2} m_1 m_2 \sqrt{r^2 - R^2}/r \).

(b) \( F = \nabla (m_2 \cdot B) = (m_2 \cdot \nabla) B = (m_2 \frac{d}{dz}) \left[ \frac{2\pi}{4\pi} \left( 3(m_1 \cdot \hat{z}) \hat{z} - m_1 \right) \right] = \frac{2\pi}{4\pi} m_1 m_2 \frac{dz}{dz} \left[ (\hat{z} \cdot \hat{z}) \right] \),

or, since \( z = r \):

\[ F = -\frac{3\mu_0 m_1 m_2}{2\pi \ r^4} \hat{z}. \]