CHAPTER 23 – Electric Potential

2. We find the work done by an external agent from the work-energy principle:
   \[ W = \Delta K + \Delta U = 0 + q(V_b - V_a) \]
   \[ = (1.60 \times 10^{-19} \text{ C}) [(-50 \text{ V}) - (+100 \text{ V})] = -2.40 \times 10^{-17} \text{ J} \text{ (done by the field)}. \]

7. For the uniform electric field between two large, parallel plates, we have
   \[ E = \frac{\Delta V}{d}; \]
   \[ 640 \text{ V/m} = \Delta V/(11.0 \times 10^{-3} \text{ m}), \]
   which gives \[ \Delta V = 7.04 \text{ V}. \]

11. The potential difference between two points in an electric field is found from
    \[ \Delta V = - \int \mathbf{E} \cdot d\mathbf{l}. \]
    (a) For \( V_{BA} \) we have
    \[ V_{BA} = - \int_{A}^{B} \mathbf{E} \cdot d\mathbf{l} = - \int_{A}^{B} (\mathbf{300 N/C}) \mathbf{i} \cdot d\mathbf{y} = 0. \]
    (b) For \( V_{CB} \) we have
    \[ V_{CB} = - \int_{A}^{C} \mathbf{E} \cdot d\mathbf{l} = - \int_{A}^{C} (\mathbf{300 N/C}) \mathbf{i} \cdot d\mathbf{x} = \int_{4 \text{ m}}^{3 \text{ m}} (\mathbf{300 N/C}) \mathbf{d}x = -2100 \text{ V}. \]
    (c) For \( V_{CA} \) we have
    \[ V_{CA} = - \int_{A}^{C} \mathbf{E} \cdot d\mathbf{l} = - \int_{A}^{C} (\mathbf{300 N/C}) \mathbf{i} \cdot (\mathbf{x} \mathbf{i} + d\mathbf{y}) = \int_{4 \text{ m}}^{3 \text{ m}} (\mathbf{300 N/C}) \mathbf{d}x = -2100 \text{ V}. \]
    Note that \( V_{CA} = V_{CB} + V_{BA} \).

14. (a) The potential at the surface of a charged sphere in terms of the charge density is
    \[ V = \frac{Q}{4\pi \varepsilon_0 r} = \frac{\sigma 4\pi r^2}{4\pi \varepsilon_0} = \frac{\sigma r}{\varepsilon_0}; \]
    \[ 500 \text{ V} = \sigma (0.16 \text{ m})/(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2), \]
    which gives \( \sigma = 2.8 \times 10^8 \text{ C/m}^2. \)
    (b) If we form the ratio for the two distances, we have
    \[ V_1/V_0 = r_0/r_1; \]
    \[ (10 \text{ V})/(500 \text{ V}) = (0.16 \text{ m})/r_1, \]
    which gives \( r_1 = 8.0 \text{ m}. \)

15. (a) After the connection, if the two spheres were at different potentials, there would be a flow of charge in the wire. Thus the potentials must be the same.
    (b) We assume the spheres are so far apart that the potential of one sphere at the other sphere is essentially zero. The initial potentials are
    \( V_{a1} = Q/4\pi \varepsilon_0 r_1, \)
    \( V_{a2} = 0. \)
    After the connection, \( Q \) is transferred to the second sphere, so we have
    \( V_a = (Q - Q)/4\pi \varepsilon_0 r_1 = V_1 = Q/4\pi \varepsilon_0 r_1, \)
    or
    \( r_1(Q - Q) = r_1Q, \)
    which gives \( Q = r_1Q/r_1 + r_2. \)

16. The radial electric field of the long wire is
    \[ E = \frac{\lambda}{2\pi \varepsilon_0 r}. \]
    We find the potential difference from
    \[ V_b - V_a = - \int_{R_a}^{R_b} \mathbf{E} \cdot d\mathbf{l} = - \int_{R_a}^{R_b} \mathbf{E} \cdot d\mathbf{r} = - \int_{R_a}^{R_b} \frac{\lambda}{2\pi \varepsilon_0 r} \mathbf{d}r = - \frac{\lambda}{2\pi \varepsilon_0} \ln \frac{R_b}{R_a} = \frac{\lambda}{2\pi \varepsilon_0} \ln \frac{R_a}{R_b}. \]

20. (a) In each region the electric field is the same as that of a point charge equal to the net charge within a spherical surface. Thus we have
    \[ E = \frac{(+Q + \frac{\lambda}{2}Q)4\pi \varepsilon_0 r^2}{4\pi \varepsilon_0 r^2} = 3Q/8\pi \varepsilon_0 r^2, r > r_1. \]
    \[ E = 0, r_1 < r < r_2 \text{ (inside a conductor), which means there is a negative charge } -\frac{\lambda}{2}Q \text{ on the inner surface; thus a positive charge } 3/2 Q \text{ on the outer surface}; \]
\[ E = \frac{1}{2} \frac{Q}{4\pi \varepsilon_0 r^2} = \frac{Q}{8\pi \varepsilon_0 r^2}, 0 < r < r_1. \]

23. We find the electric potentials of the stationary charges at the initial and final points:

\[ V_a = (1/4\pi \varepsilon_0) \left( \frac{Q}{r_a} + \frac{Q}{r_b} \right) \]
\[ = \left( 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C} \right) \left\{ \left[ \frac{25 \times 10^4 \text{ C}}{0.030 \text{ m}} \right] + \left[ \frac{25 \times 10^4 \text{ C}}{0.030 \text{ m}} \right] \right\} = 1.50 \times 10^8 \text{ V}. \]

\[ V_b = (1/4\pi \varepsilon_0) \left( \frac{Q}{r_a} + \frac{Q}{r_b} \right) \]
\[ = \left( 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C} \right) \left\{ \left[ \frac{25 \times 10^4 \text{ C}}{0.040 \text{ m}} \right] + \left[ \frac{25 \times 10^4 \text{ C}}{0.020 \text{ m}} \right] \right\} = 1.69 \times 10^8 \text{ V}. \]

Because there is no change in kinetic energy, we have

\[ W_{a \rightarrow b} = \Delta K + \Delta U = 0 + q(V_b - V_a) \]
We find the electric potentials of the stationary charges at the initial and final points:

\[
V_a = \frac{1}{4\pi \varepsilon_0}\left(\frac{Q_1}{r_{1a}} + \frac{Q_2}{r_{2a}}\right)
= (9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)\left\{\frac{(25 \times 10^{-6} \text{ C})}{0.030 \text{ m}} + \frac{(25 \times 10^{-6} \text{ C})}{0.030 \text{ m}}\right\} = 1.50 \times 10^7 \text{ V}.
\]

\[
V_b = \frac{1}{4\pi \varepsilon_0}\left(\frac{Q_1}{r_{1b}} + \frac{Q_2}{r_{2b}}\right)
= (9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)\left\{\frac{(25 \times 10^{-6} \text{ C})}{0.040 \text{ m}} + \frac{(25 \times 10^{-6} \text{ C})}{0.020 \text{ m}}\right\} = 1.69 \times 10^7 \text{ V}.
\]

Because there is no change in kinetic energy, we have

\[
W_{a \rightarrow b} = \Delta K + \Delta U = 0 + q(V_b - V_a) = (0.10 \times 10^{-6} \text{ C})(1.69 \times 10^7 \text{ V} - 1.50 \times 10^7 \text{ V}) = +0.19 \text{ J}.
\]

27. When the electron is far away, the potential from the fixed charge is zero. Because energy is conserved, we have

\[
\Delta K + \Delta U = 0; \quad m v^2 - 0 + (- e)(0 - V) = 0, \quad \text{or} \quad m v^2 = -e\left(\frac{kQ}{r}\right) = 9.11 \times 10^{-31} \text{ kg} v^2 = - (1.60 \times 10^{-19} \text{ C})(9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-0.125 \times 10^{-6} \text{ C})/(0.725 \text{ m}),
\]

which gives \( v = 2.33 \times 10^7 \text{ m/s} \).

30. For the potential at point A we have

\[
V_a = \frac{1}{4\pi \varepsilon_0}\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right)
= \frac{Q}{4\pi \varepsilon_0}(3/L + 1/L\sqrt{2} - 2/L)
= \frac{(1 + 1/\sqrt{2})Q}{4\pi \varepsilon_0 L}.
\]

31. We choose a ring of radius \( r \) and width \( dr \) for a differential element, with charge \( dq = \sigma 2\pi r dr \). The potential of this element on the axis a distance \( x \) from the ring is

\[
dV = dq/4\pi \varepsilon_0(x^2 + r^2)^{3/2}
= \sigma 2\pi r dr/4\pi \varepsilon_0(x^2 + r^2)^{3/2} = \sigma r dr/2 \varepsilon_0(x^2 + r^2)^{3/2}.
\]

We integrate to get the potential:

\[
V = \int_{R_1}^{R_2} \frac{\sigma r dr}{2\varepsilon_0 \left(x^2 + r^2\right)^{1/2}} = \frac{\sigma}{2\varepsilon_0} \left[\left(x^2 + R_2^2\right)^{1/2} - \left(x^2 + R_1^2\right)^{1/2}\right].
\]

47. From the spatial dependence of the electric potential, \( V(x, y, z) = y^2 + 2xy - 4xyz \), we find the components of the electric field from the partial derivatives of \( V \):

\[
E_x = -\partial V/\partial x = -(2y - 4yz);
E_y = -\partial V/\partial y = -(2y + 2x - 4xz);
E_z = -\partial V/\partial z = -(4xy).
\]

We can write the electric field: \( \mathbf{E} = 2y(2z - 1)i - 2(y + x - 2xz)j + (4xy)k \).
56. (a) For the potential energy of the four charges we have
\[
U = \frac{1}{4\pi \varepsilon_0} (Q_1 Q_2 / r_{12} + Q_1 Q_3 / r_{13} + Q_1 Q_4 / r_{14} + Q_2 Q_3 / r_{23} + Q_2 Q_4 / r_{24} + Q_3 Q_4 / r_{34})
\]
\[
= \left( \frac{Q_2}{4\pi \varepsilon_0} \right) (1/b + 1/b\sqrt{2} + 1/b + 1/b\sqrt{2} + 1/b)
\]
\[
= \left( 4 + \sqrt{2} \right) \frac{Q_2}{4\pi \varepsilon_0 b}.
\]

(b) We find the potential energy with a charge at the center from the potential of the four charges at the center:
\[
U_C = \frac{+Q}{4\pi \varepsilon_0} \frac{4Q}{b/\sqrt{2}} = \sqrt{2} \frac{Q_2}{\pi \varepsilon_0 b}.
\]

(c) To test for stability, we find the potential energy change when the charge at the center is displaced slightly toward one of the corners. If the displacement is \(\delta \ll b\), we have
\[
U = \left( \frac{\sqrt{2}Q_2}{4\pi \varepsilon_0 b} \right) \left\{ \frac{2}{[1 + (\sqrt{2}\delta b)^2]^{1/2}} + \frac{1}{[1 + (\sqrt{2}\delta b)]} + \frac{1}{[1 - (\sqrt{2}\delta b)]} \right\}
\]
\[
= \left( \frac{\sqrt{2}Q_2}{4\pi \varepsilon_0 b} \right) \left\{ 2[1 + (\sqrt{2}\delta b)^2]^{1/2} + 1/[1 + (\sqrt{2}\delta b)] + 1/[1 - (\sqrt{2}\delta b)] \right\}
\]
\[
= \left( \frac{\sqrt{2}Q_2}{4\pi \varepsilon_0 b} \right) \left\{ 2[1 + (\sqrt{2}\delta b)^2]^{1/2} + 2/[1 - (\sqrt{2}\delta b)] \right\}
\]
If we use the approximation \((1 \pm x)^n \approx 1 - nx\), we get
\[
U \approx \left( \frac{\sqrt{2}Q_2}{2\pi \varepsilon_0 b} \right) \left\{ 1 - \left[ \frac{1}{[1 + (\sqrt{2}\delta b)]} + 1/[1 - (\sqrt{2}\delta b)] \right] \right\}
\]
\[
= \left( \frac{\sqrt{2}Q_2}{2\pi \varepsilon_0 b} \right) \left\{ 1 - \left[ \frac{1}{1 + (\sqrt{2}\delta b)^2} + 1 + 2(\sqrt{2}\delta b)^2 \right] \right\}
\]
Thus we see that \(U > U_C\), so work would have to be done to move the fifth charge away from the center. The fifth charge is in stable equilibrium.

(d) Because the potential at the center from the charges at the four corners does not change, the potential energy of the fifth charge will be
\[
U_c = \left( -\frac{Q}{4\pi \varepsilon_0 b} \right) \left\{ \frac{4Q}{\sqrt{2}} \right\} = -\sqrt{2} \frac{Q_2}{\pi \varepsilon_0 b}.
\]
If we again consider a small displacement from the center, the new potential energy will be
\[
U = -\sqrt{2} \frac{Q_2}{\pi \varepsilon_0 b} \left[ 1 + 2(\sqrt{2}\delta b)^2 \right].
\]
Thus \(U < U_C\), so the charge will acquire kinetic energy if it moves away from the center. The fifth charge is in unstable equilibrium.

The maximum kinetic energy will be acquired when the fifth charge reaches one of the corners. Because the potential goes to \(3/2\) there, the maximum kinetic energy would be \(\infty\).
This is due to the idealization of point charges.